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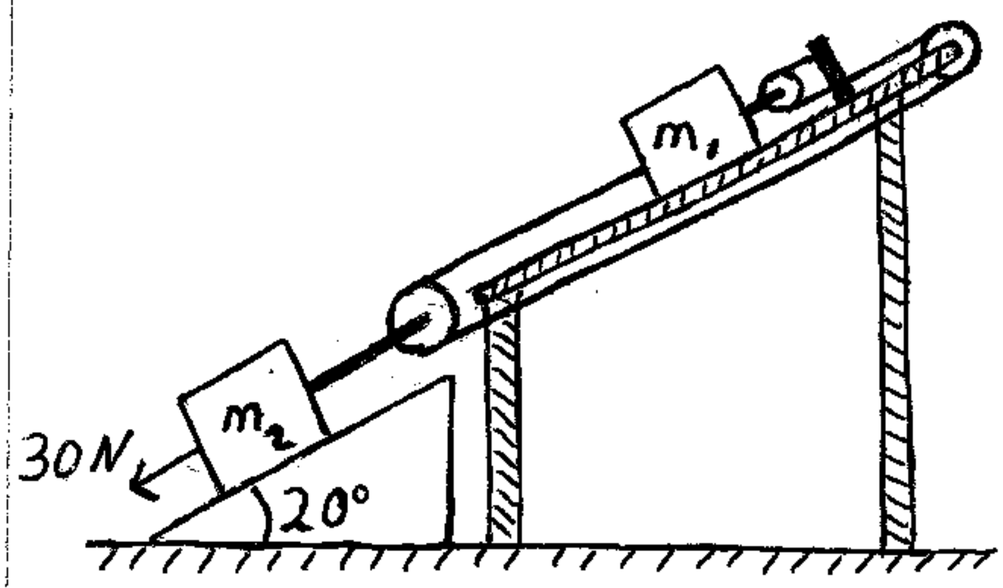
This is a force and motion problem involving pulleys.

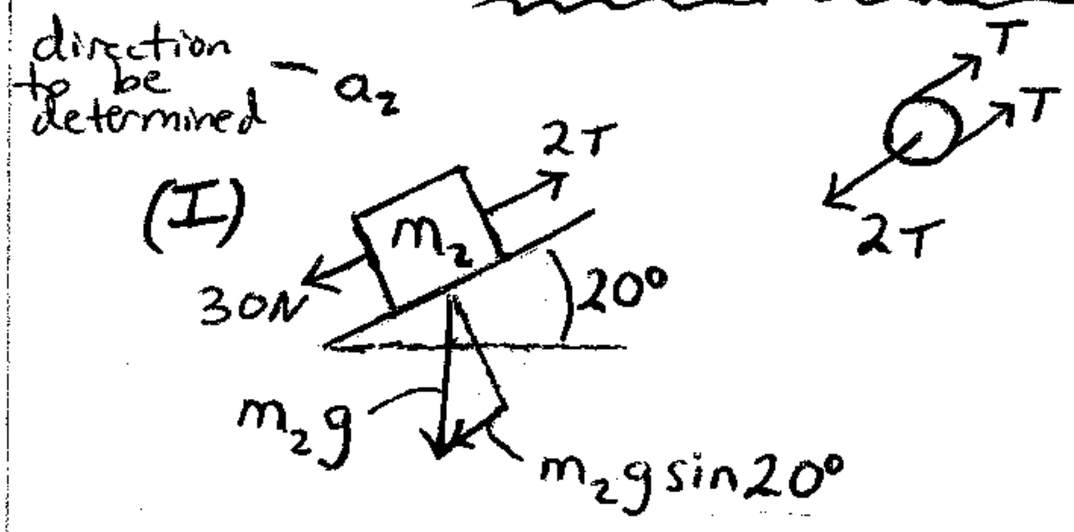


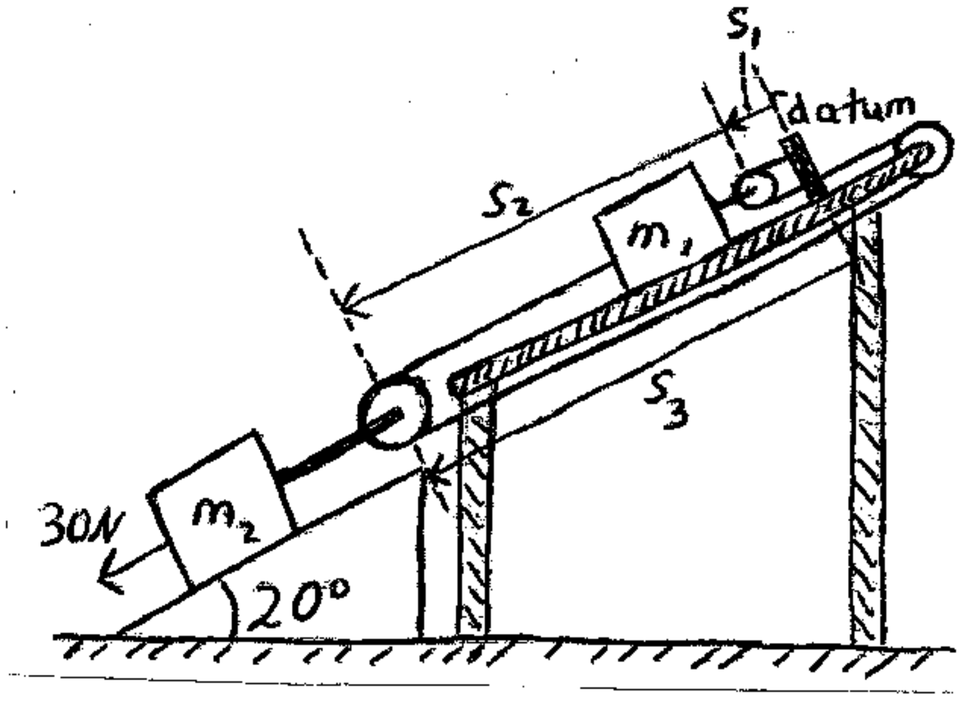
In the pulley system shown, two blocks are released from rest. The sliding surfaces are frictionless, and the mass of the pulleys and rope are negligible. After 1.3 s, what is the velocity of each block?

Note that $m_2 = 30\,kg$ and $m_1 = 10\,kg$.

Solution:

Free-body diagrams





constant length

$2s_1 + s_3 + s_2 = L$ (1)

$s_2 = s_3 - s_1$ (2)

Combine equations (1) and (2):

$$2s_1 + s_3 + s_3 - s_1 = L$$

$$\Rightarrow \quad s_1 + 2s_3 = L$$

differentiate twice with respect to time:

$$a_1 + 2a_3 = 0$$

Since a_3 is actually equal to the acceleration of mass m_2, then $a_3 = a_2$.

$$\Rightarrow a_1 + 2a_2 = 0 \quad (3)$$

(I) Apply Newton's second law:

$$30 + m_2 g \sin 20° - 2T = m_2 a_2 \quad (4)$$

(II) Apply Newton's second law:

$$m_1 g \sin 20° + T - 2T = m_1 a_1 \quad (5)$$

Substitute given values:

$$(3) \Rightarrow \quad a_1 + 2a_2 = 0$$

$$(4) \Rightarrow \quad 30 + 30(9.8)\sin 20° - 2T = 30 a_2$$

$$(5) \Rightarrow \quad 10(9.8)\sin 20° - T = 10 a_1$$

Combine equations (3)-(5) and solve:

$a_1 = -1.814 \ m/s^2 \Rightarrow V_{f1} = 0 - 1.814(1.3) = -2.36 \ m/s$

$a_2 = 0.907 \ m/s^2 \Rightarrow V_{f2} = 0 + 0.907(1.3) = 1.18 \ m/s$

$T = 51.666 \ N$ (ans.)