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If someone can vertically jump a height h_1 on earth with $g = 9.8\,m/s^2$, how high can they jump on another planet with a different value of g? Let H be the vertical lift distance (before takeoff) of the center of mass of the jumper during the propelling phase of the jump.

Solution: The jump height h_1 corresponds to the height reached after the jumper's feet leave the ground.

To find the takeoff velocity v_1 (when the jumper's feet leave the ground) we can use the following projectile motion equation, since the only force acting on the jumper after takeoff, is gravity.

$$v_f^2 = v_1^2 - 2g\Delta d \quad , \quad g = 9.8\,m/s \text{ (on earth)}$$

At maximum jump height $\Delta d = h_1$, and $v_f = 0$. Therefore,

$$0 = v_1^2 - 2gh_1 \longrightarrow v_1^2 = 2gh_1$$

You need to know how much the center of mass of the jumper lifts vertically during the propelling phase of the jump. This is the phase of the jump where the jumper increases his vertical upward speed in order to be able to jump. The vertical lift distance is given by H. This can be assumed constant no matter what planet the jumper is on.

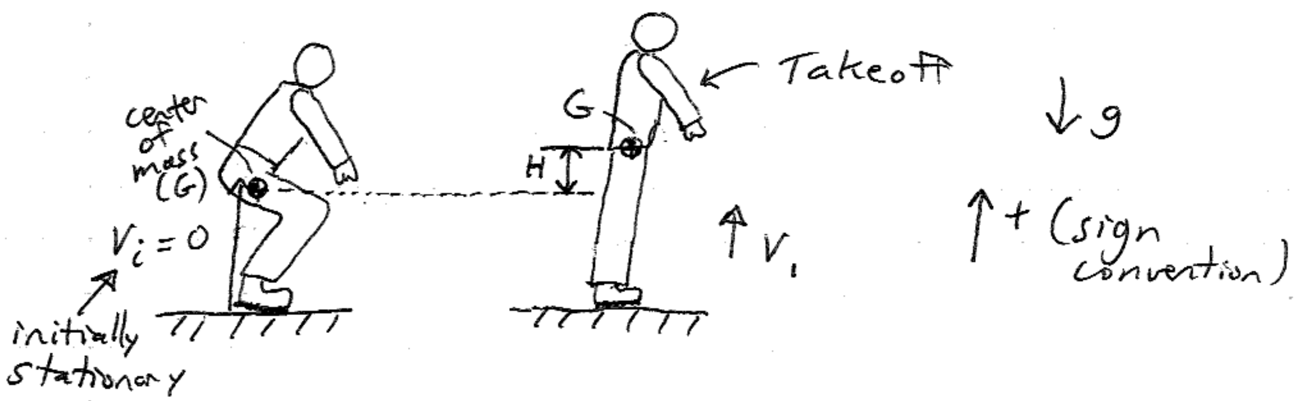
Let g_2 be the acceleration due to gravity on the other planet

Let m be the mass of the jumper

Let v_2 be the takeoff velocity of the jumper on the other planet

Let U be the internal work input of the jumper during the propelling phase of the jump. You can assume this is always constant since it represents the internal work done by the jumper's muscles. If you don't know it, look up the principle of work and energy for systems of particles, as it explains the difference between internal and external work. External work is the work done by gravity, which also acts on the jumper.

Next, apply the principle of work and energy for a system of particles



At the start of the jump (as the jumper is crouched down and stationary) the kinetic energy of the jumper is zero, since V_i (initial velocity) is zero.

Therefore, the work and energy equation for the jump on earth becomes:

$$\underbrace{\frac{1}{2} m v_i^2}_{0} - m g H + U = \frac{1}{2} m v_1^2$$

which becomes

$$-mgH + U = \frac{1}{2} m (2gh_1) \quad (1)$$

Similarly, the work and energy equation for the jump on another planet is given by:

$$-m g_2 H + U = \frac{1}{2} m (2 g_2 h_2) \quad (2)$$

where h_2 is the jump height on the other planet.

Eliminate U between (1) and (2) and solve for h_2. We get:

$$\cancel{m} g H - \cancel{m} g_2 H = \cancel{m} g_2 h_2 - \cancel{m} g h_1$$

$$h_2 = \frac{gH - g_2 H + g h_1}{g_2}$$

For example, let's say $H = 0.5\,m$, $h_1 = 0.3\,m$, and $g_2 = 1.6\,m/s^2$ (on the moon).

Then $h_2 = 4.4\,m$ – this is much higher!

Note that, upon solving for U in equation (1), and substituting into (2), if the left side of (2) < 0 then the jumper is unable to jump off the planet, since the gravity is too high!