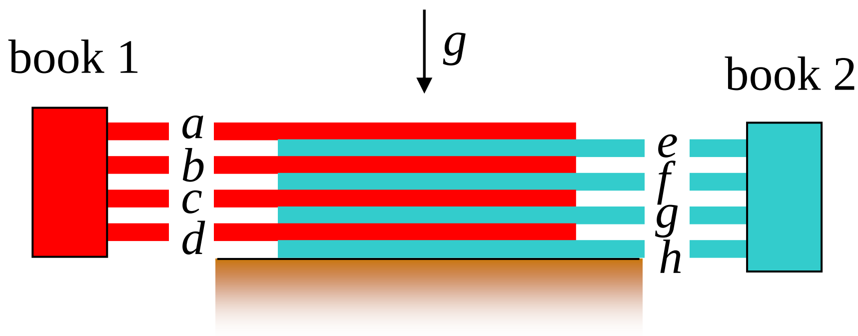
Are two interlaced phone books impossible to pull apart by any means?

The phone books have to be sitting on a surface so that the weight of the pages is felt throughout the thickness. This will maximize the friction force. To analyze this problem consider the figure below showing four interlaced pages from two books. From this result we will generalize.



Let's say the weight of each page is W and the coefficient of static friction between the pages is μ_s.

To calculate the total friction force holding the books together we have to add up the individual friction forces acting on all the page surfaces that are in contact with other page surfaces.

Let's start from the topmost page and work down.

The weight of page a is W so the friction force between a and e is $W\mu_s$.

The combined weight of pages a and e is $2W$ so the friction force between b and e is $2W\mu_s$.

The combined weight of pages a, e, b is $3W$ so the friction force between b and f is $3W\mu_s$.

The combined weight of pages a, e, b, f is $4W$ so the friction force between c and f is $4W\mu_s$.

The combined weight of pages a, e, b, f, c is $5W$ so the friction force between c and g is $5W\mu_s$.

The combined weight of pages a, e, b, f, c, g is $6W$ so the friction force between d and g is $6W\mu_s$.

The combined weight of pages a, e, b, f, c, g, d is $7W$ so the friction force between d and h is $7W\mu_s$.

The total friction force holding the books together is $F_{TOT} = W\mu_s + 2W\mu_s + 3W\mu_s + 4W\mu_s + 5W\mu_s + 6W\mu_s + 7W\mu_s = W\mu_s(1+2+3+4+5+6+7) = 28W\mu_s$. From this result we can generalize for two identical interlaced books, each having N number of pages, and where each page has weight W:

$F_{TOT} = W\mu_s(1+2+3+...+2N-1) = W\mu s(N)(2N-1)$, where $(1+2+3+...+2N-1) = (2N)(2N-1)/2 = (N)(2N-1)$ from math class.

Let's say we have a phone book that weighs 2.5 pounds and has $N = 1000$ pages. What is the friction force holding two of these interlaced phone books together?

Each page has weight $W = 0.0025$ pounds (equal to 2.5/1000). Let's say $\mu_s = 0.2$.

Using the above formula we have:

$F_{TOT} = 0.0025(0.2)(1000)(2\text{x}1000-1) = 1000$ pounds

This is an incredible amount of force required to pull these two books apart!

Can you use a whip to swing safely across a chasm? (from Indiana Jones movie)

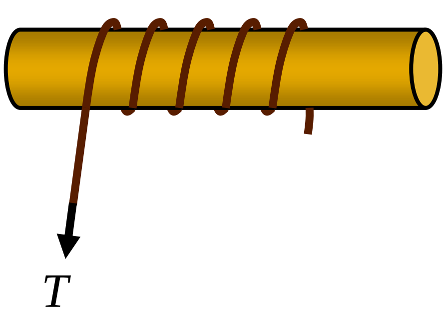
Yes, provided the whip is wound around a pole or branch enough times. The friction force between pole (or branch) and whip will increase with the number of windings.

This phenomenon can be described mathematically.

As an example, consider the following problem.

A rope is wrapped around a pole of radius $R = 3$ cm. If the tension on one end of the rope is $T = 1000$ N, and the coefficient of static friction between the rope and pole is $\mu = 0.2$, what is the minimum number of times the rope must be wrapped around the pole so that it doesn’t slip off?

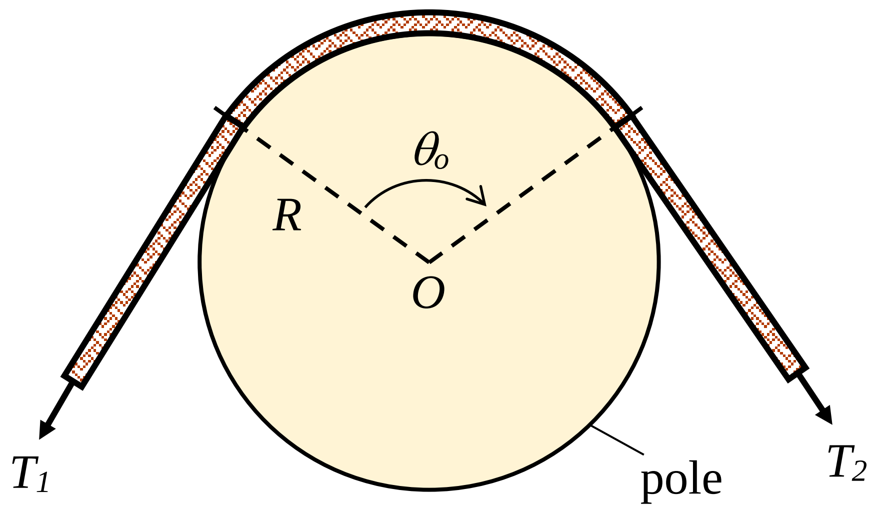
Assume that the minimum number of times the rope must be wrapped around the pole corresponds to a tension of 1 N on the other end of the rope.



Solution**:**

To solve this problem we have to derive an expression for the rope tension around the pole, using Calculus.

To start, consider a general case as illustrated below.



Where:

T_1 and T_2 is the rope tension on both ends of the rope

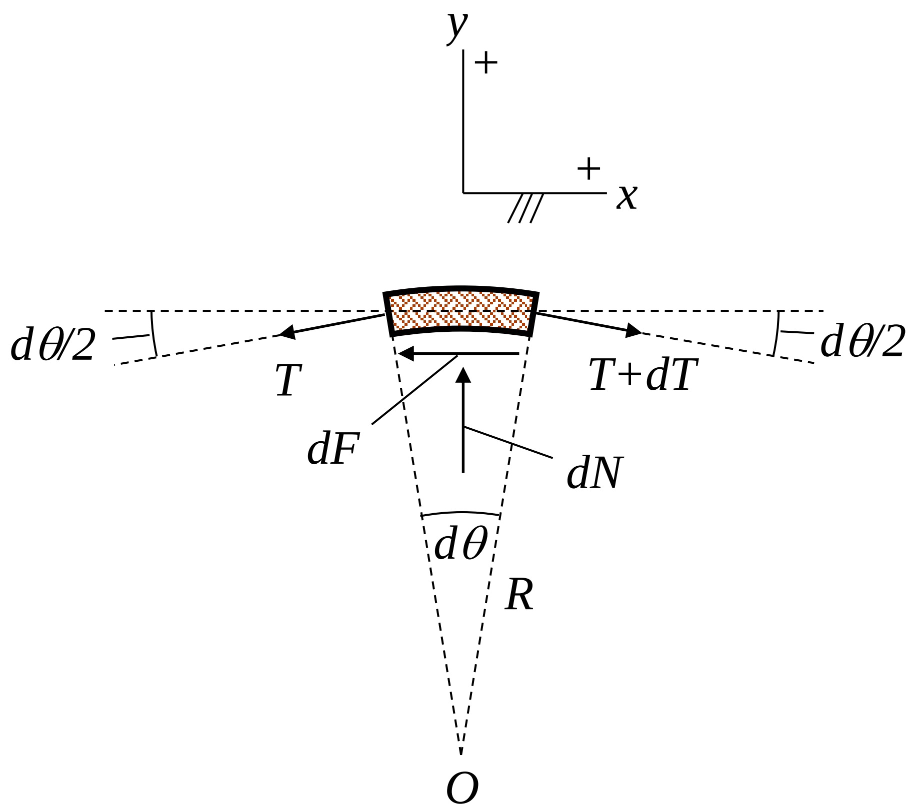
θ_o is the angle the rope is wrapped around the pole, as shown (in radians)

R is the radius of the pole

O is the center of the pole

Next, derive a general expression for the rope tension as a function of T_1, T_2, θ_o, and R.

Consider a differential segment of rope, illustrated below. Treat this as a two-dimensional problem in the *xy* plane.



Where:

dN is the differential normal force between the pole and differential rope segment

dF is the differential friction force between the pole and differential rope segment

T is the rope tension

$d\theta$ is the differential angle spanned by the differential section of rope (in radians)

Since the differential rope segment is in static equilibrium, the sum of the forces acting on it in the *xy* plane is equal to zero.

In the *x*-direction, take the sum of the horizontal forces and equate them to zero:

$$(T+dT)\cos\left(\frac{d\theta}{2}\right)-T\cos\left(\frac{d\theta}{2}\right)-dF=0 \qquad (1)$$

where

$$dF = \mu dN \qquad (2)$$

Similarly, in the *y*-direction, take the sum of the vertical forces and equate them to zero:

$$dN - T\sin\left(\frac{d\theta}{2}\right) - (T + dT)\sin\left(\frac{d\theta}{2}\right) = 0 \qquad (3)$$

Combine equations (1), (2), (3) and take the limit as $d\theta \to 0$. This gives us

$$\frac{dT}{d\theta} = \mu T$$

We can rewrite this as

$$\frac{dT}{T} = \mu d\theta$$

Integrate both sides of this equation and solve for *T* as a function of θ. We get

$$T = Ce^{\mu\theta}$$

where *C* is a constant.

At $\theta = 0$, $T = T_1$, which means that $C = T_1$.

Thus,

$$T_2 = T_1 e^{\mu\theta}$$

It is interesting that this equation does not depend on the radius *R* of the pole. But this is perhaps not too surprising since *R* does not show up in equations (1), (2), (3).

Set $\theta = \theta_o$ in order to remain consistent with the variables shown in the figure on page 3.

Therefore, the final equation is

$$T_2 = T_1 e^{\mu\theta_o}$$

If we assume that $T_2 < T_1$ then we must set $\mu < 0$, since the direction of static friction depends on which direction the rope will tend to slide. This in turn depends on the relative magnitude of T_1 and T_2.

Similarly, if we assume that $T_2 > T_1$ then we must set $\mu > 0$.

In our case, assume that T_1 = 1000 N, and T_2 = 1 N. This means that we must set μ = -0.2. Using the above equation solve for θ_o.

Solving, we get θ_o = 34.54 radians. This is equal to 5.5 turns, which is the minimum number of times the rope must be wrapped around the pole to prevent it from slipping off. Since 1000 N = 102 kg, and Indiana Jones is probably quite a bit lighter than this, then wrapping the whip around a branch 5-6 times should be enough to support his weight, even with the extra bit of weight created by his centripetal acceleration as he swings along an arc.