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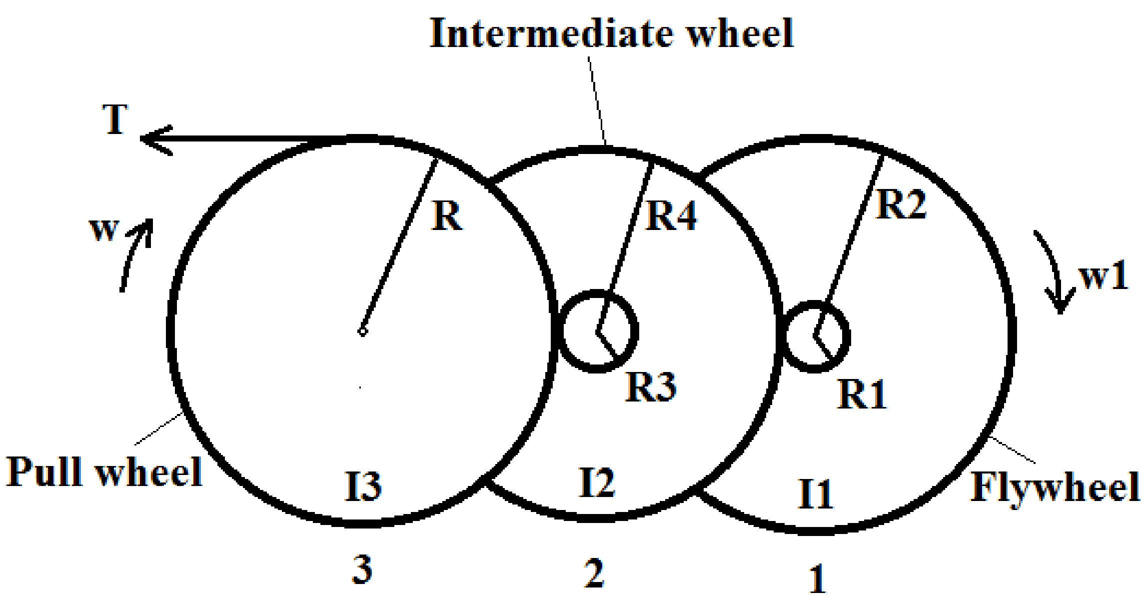
Pulling Heavy Payload With Flywheel

A heavy payload of mass 5000 kg needs to be pulled along a flat surface that has rollers on it. The speed of the payload as it is pulled is not to exceed 20 m/s. The payload is to increase in speed gradually as it is pulled, starting from rest. The pulling distance is 20 m. The plan is to use an engine or motor to rotate a heavy flywheel up to a certain rotational speed, and then disengage the engine/motor and use the rotating flywheel to pull the payload. Doing this avoids having to use a very powerful engine/motor, and instead allows the use of a much less powerful engine/motor to slowly add energy to an inertial system which will then have enough power to pull the payload.

Design a flywheel system that can pull the payload as desired.

Solution:

The figure below shows the set up containing three wheels, which rotate together using gears. The figure is labelled with different variable names.



If you have a fast spinning flywheel with two gears attached you can generate a very large pulling force. The pull wheel shown on the left is what pulls the payload. The payload is pulled towards the right, with a rope, by the pull wheel which rotates clockwise. The fast spinning flywheel shown on the right provides the energy for the pull (this is the flywheel rotated up to speed by the engine/motor). The three wheels contain gears and they are arranged together to convert the flywheel rotation into a large torque for pulling the payload (using the pull wheel). The flywheel has to be rotated to a high speed using a motor. The variables in the figure are as follows:

w1 = angular velocity of the flywheel (clockwise)

R1 = radius of the small gear attached to the flywheel. This gear turns wheel/gear 2.

R2 = outer radius of the flywheel

I1 = rotational inertia of the flywheel

R3 = radius of the small gear attached to the intermediate wheel/gear. This gear turns wheel/gear 3. This is the pull wheel.

R4 = outer radius of the intermediate wheel/gear

I2 = rotational inertia of the intermediate wheel/gear

R = outer radius of the pull wheel

I3 = rotational inertia of the pull wheel/gear

T = tension in the pull wheel rope

w = angular velocity of the pull wheel/gear (clockwise)

The following quantities are used: Flywheel mass = 50 kg, initial flywheel clockwise rotation speed = 200 rad/s, R1 = 0.05 m, R2 = 0.50 m, R3 = 0.05 m, R4 = 0.50 m, R = 0.20 m. This produces an initial pull wheel clockwise rotation speed = 5 rad/s. This results on the pull wheel pulling the payload at 0.20*5 = 1 m/s initially (this is the maximum speed, and the speed slows down as the pull progresses, because the flywheel slows down). You can adjust the wheel/gear dimensions as you desire to get the pulling speed that you want.

The angular velocity of the flywheel and pull wheel are related by this equation:

w1 = w*(R4/R3)*(R/R1)

The initial energy of the flywheel must be at least equal to the energy it takes to pull the payload. If we assume the flywheel is a solid cylinder, then its rotational inertia is equal to (1/2)*50*(R2)^2 = 6.25 kg-m^2. The rotational energy of the flywheel is (1/2)*6.25*200^2 = 125000 Joules. The energy to pull the payload = (pulling force)*(pulling distance). Here I use a coefficient of rolling friction of 0.1. Hence, pulling force = (mass of payload)*9.8*0.1 = 4900, where 9.8 m/s^2 is the acceleration due to gravity, and we are assuming the acceleration of the payload is negligible during the pull. The pulling distance is 20 meters. Therefore, the energy to pull the payload is equal to 4900*20 = 98000 Joules. This is less than the flywheel energy of 125000 Joules so there is enough flywheel energy to pull the payload a distance of 20 m.

My calculations are as follows:

I use the sign convention of clockwise as negative and counterclockwise as positive.

The torque equation for the flywheel is:

F1*R1 = I1*alpha1   (equation 1)

where:

F1 is the tangential contact force between the small gear on the flywheel and the intermediate wheel/gear

alpha1 is the angular acceleration of the flywheel

The torque equation for the intermediate wheel/gear is:

F1*R4 - F2*R3 = I2*alpha2   (equation 2)

where:

F2 is the tangential contact force between the small gear on the intermediate wheel/gear and the pull wheel

alpha2 is the angular acceleration of the intermediate wheel/gear

The torque equation for the pull wheel is:

T*R - F2*R = I3*alpha3   (equation 3)

where alpha3 is the angular acceleration of the pull wheel

We can relate the angular acceleration of the wheels/gears as follows:

alpha1*R1 = -R4*alpha2   (equation 4)

alpha3*R = -R3*alpha2   (equation 5)

Combine equations 1-5 to obtain the final torque equation for the pull wheel which relates the tension (T) pulling on the pull wheel with the angular acceleration of the pull wheel (alpha3):

T*R = Ip*alpha3

where Ip is the effective rotational inertia of the pull wheel. This means that the three-wheel assembly is effectively the same as a single pull wheel with rotational inertia Ip, where Ip is given by:

Ip = I3 + I2*(R/R3)^2 + I1*(R*R4)^2/(R1*R3)^2

For the quantities I used, the last term dominates in this above equation, so to a good approximation:

Ip ≈ I1*(R*R4)^2/(R1*R3)^2

If we assume the flywheel is a solid cylinder, then I1 = (1/2)*m1*(R2)^2, where m1 is the mass of the flywheel.

As a result,

Ip = (1/2)*me*R^2

where

me = m1*(R2*R4)^2/(R1*R3)^2

This is the effective mass of the pull wheel (if we assume it is a solid cylinder). Notice that it is much, much higher than the mass of the flywheel (m1).

These equations can be used for any case that you wish depending on the mass of payload you wish to pull, the maximum pulling speed, etc. You just adjust the variables to get what you want.

If you don't want the payload to be "yanked" initially, and speed up gradually, you can attach the pulling rope to one end of a strong spring and the other end of the spring is attached to the payload. This way the pulling force is built up slowly on the payload, as the spring stretches. I didn't account for a spring in these calculations but I don't think the results would change much.