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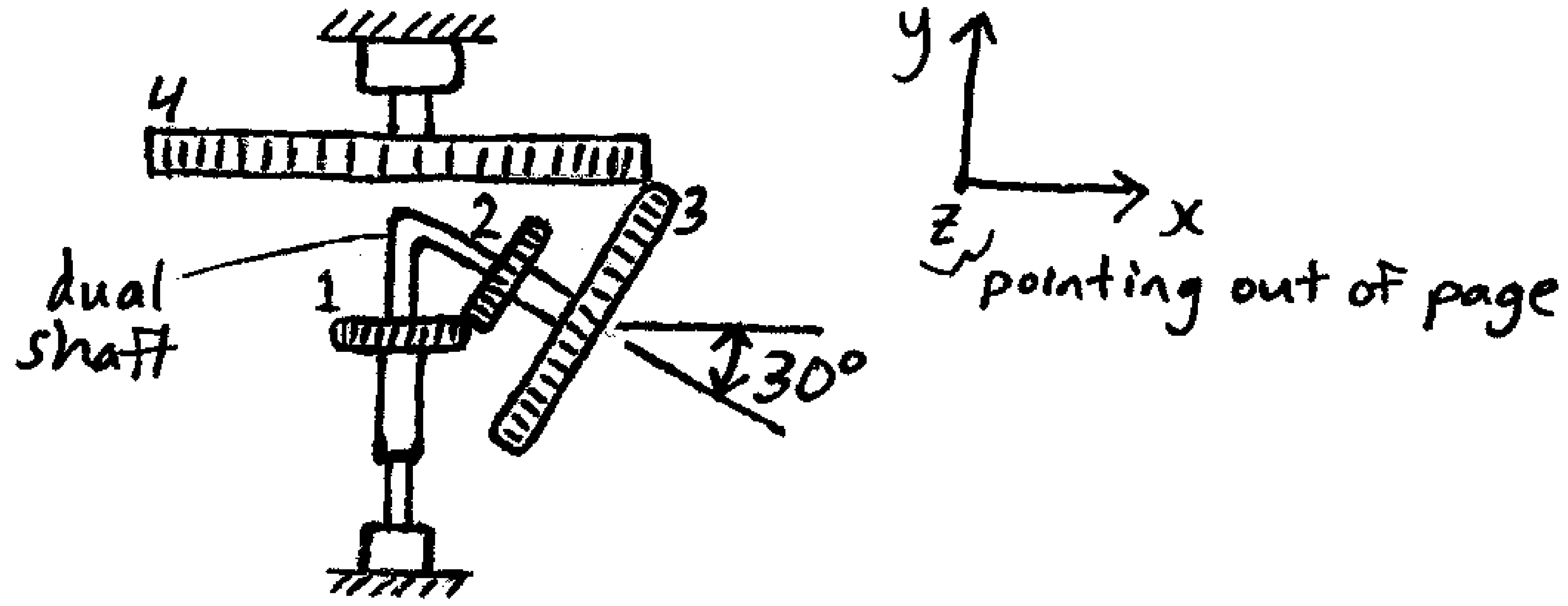
This is a 3D general motion problem (engineering mechanics).



In the gear system shown, gear 4 rotates with an angular velocity of ω_4 and an angular acceleration of α_4, with rotation vector pointing along the negative y-direction. Gear 1 rotates with an angular velocity of ω_1 and an angular acceleration of α_1, with rotation vector pointing along the negative y-direction. Gears 2 and 3 are rigidly connected to each other, and rotate freely about the dual shaft shown. Gear 1 also rotates freely about the dual shaft shown. Let R_1, R_2, R_3, R_4 be the radius of all the gears shown. Determine the angular velocity and angular acceleration of the dual shaft and gears 2 and 3.

Solution:

Let P_1 be the contact point between gear 1 and gear 2. Let A be the point on the dual shaft which is in contact with gear 1 - this is the center of gear 1.

Analyze gear 1:

$$\vec{v}_{P_1} = \underset{\substack{\downarrow \\ 0}}{\vec{v}_A} + \vec{w}_1 \times \vec{r}_{P_1/A} \quad , \quad \vec{w}_1 = -w_1 \hat{j}$$

$$\vec{r}_{P_1/A} = R_1 \hat{i}$$

$$\vec{v}_{P_1} = -w_1 \hat{j} \times R_1 \hat{i}$$

$$\vec{v}_{P_1} = w_1 R_1 \hat{k} \qquad (1)$$

Now, let B be the point on the dual shaft which is in contact with gear 2 - this is the center of gear 2.

Analyze gear 2:

$$\vec{v}_{P_1} = \vec{v}_B + \vec{w}_2 \times \vec{r}_{P_1/B}$$

Let $\vec{w}_{ds}$ and $\vec{\alpha}_{ds}$ be the angular velocity and angular acceleration of the dual shaft, respectively. Let $\vec{w}_s$ and $\vec{\alpha}_s$ be the spin angular velocity and angular acceleration of gear 2, respectively. (and gear 3)

Then, $\vec{w}_{ds} = w_{ds} \hat{j}$, $\vec{\alpha}_{ds} = \alpha_{ds} \hat{j}$

$$\vec{r}_{P_1/B} = -R_2 \sin 30^\circ \hat{i} - R_2 \cos 30^\circ \hat{j}$$

Now, $\vec{v}_B = \underset{\substack{\downarrow \\ 0}}{\vec{v}_A} + \vec{w}_{ds} \times \vec{r}_{B/A}$

$$\vec{r}_{B/A} = (R_1 + R_2 \sin 30^\circ)\hat{i} + R_2 \cos 30^\circ \hat{j}$$

Then,

$$\vec{w}_2 = \vec{w}_{ds} + \vec{w}_s \quad , \quad \vec{w}_s = w_s \cos 30^\circ \hat{i} - w_s \sin 30^\circ \hat{j}$$

$$\vec{v}_{P_1} = w_{ds}\hat{j} \times [(R_1 + R_2 \sin 30^\circ)\hat{i} + R_2 \cos 30^\circ \hat{j}]$$
$$+ (w_{ds}\hat{j} + w_s \cos 30^\circ \hat{i} - w_s \sin 30^\circ \hat{j})$$
$$\times (-R_2 \sin 30^\circ \hat{i} - R_2 \cos 30^\circ \hat{j})$$

$$\vec{V}_{P_1} = -w_{ds}(R_1 + R_2 \sin 30°)\hat{k} + (w_{ds} - w_s \sin 30°)R_2 \sin 30°\hat{k}$$
$$- w_s \cos 30° R_2 \cos 30°\hat{k}$$

$$\vec{V}_{P_1} = \Big[-w_{ds}(R_1 + R_2 \sin 30°) + (w_{ds} - w_s \sin 30°)R_2 \sin 30°$$
$$- w_s R_2 (\cos 30°)^2\Big]\hat{k} \qquad (2)$$

Let P_2 be the contact point between gear 3 and gear 4. Let C be the point on the dual shaft which is in contact with gear 3 - this is the center of gear 3. Let d be the distance between gear 2 and gear 3.

Analyze gear 3:

$$\vec{V}_{P_2} = \vec{V}_C + \vec{w}_3 \times \vec{r}_{P_2/C} \qquad (\text{note: } \vec{w}_2 = \vec{w}_3)$$

$$\vec{V}_C = \underset{0}{\vec{V}_A} + \vec{w}_{ds} \times \vec{r}_{C/A}$$

$$\vec{r}_{P_2/C} = R_3 \sin 30°\hat{i} + R_3 \cos 30°\hat{j}$$

$$\vec{r}_{C/A} = (R_1 + R_2 \sin 30° + d\cos 30°)\hat{i}$$
$$+ (R_2 \cos 30° - d \sin 30°)\hat{j}$$

$$\vec{w}_3 = \vec{w}_{ds} + \vec{w}_s$$

$$\vec{V}_{P_2} = w_{ds}\hat{j} \times \left[(R_1 + R_2 \sin 30° + d\cos 30°)\hat{i} + (R_2 \cos 30° - d\sin 30°)\hat{j}\right]$$
$$+ (w_{ds}\hat{j} + w_s \cos 30°\hat{i} - w_s \sin 30°\hat{j})$$
$$\times (R_3 \sin 30°\hat{i} + R_3 \cos 30°\hat{j})$$

$$\vec{V}_{P_2} = -w_{ds}(R_1 + R_2\sin 30° + d\cos 30°)\hat{k}$$
$$- (w_{ds} - w_s\sin 30°)R_3\sin 30°\hat{k}$$
$$+ w_s R_3(\cos 30°)^2\hat{k}$$

$$\vec{V}_{P_2} = [-w_{ds}(R_1 + R_2\sin 30° + d\cos 30°) - (w_{ds} - w_s\sin 30°)R_3\sin 30° + w_s R_3(\cos 30°)^2]\hat{k} \quad (3)$$

Analyze gear 4:

Let D be the center point of gear 4.

$$\vec{V}_{P_2} = \underbrace{\vec{V}_D}_{0} + \vec{w}_4 \times \vec{r}_{P_2/D}$$

$$\vec{V}_{P_2} = -w_4\hat{j} \times R_4\hat{i}, \quad \vec{w}_4 = -w_4\hat{j}$$

$$\vec{V}_{P_2} = w_4 R_4\hat{k} \quad (4) \qquad \vec{r}_{P_2/D} = R_4\hat{i}$$

By geometry, $R_1 + R_2\sin 30° + d\cos 30° + R_3\sin 30° = R_4$ (5)

Lastly, solve for w_{ds} and w_s.

Equate:

Equation (1) = Equation (2) (A)

Equation (3) = Equation (4) (B)

Combine equations (A), (B), and (5) and solve:

$$w_{ds} = \frac{-(w_1 R_1 R_3 + w_4 R_2 R_4)}{R_1 R_3 + R_2 R_4}$$

$$w_s = \frac{R_4}{R_3}(w_4 + w_{ds})$$

Solving for α_{ds} and α_s would result in the exact same form as equations (1)-(4), leading to the same form for α_{ds} and α_s as the above two expressions for w_{ds} and w_s. You would have to equate the tangatial acceleration (in the z-direction) for meshing gears at contact point P_1 and at contact point P_2. As a result,

$$\alpha_{ds} = \frac{-(\alpha_1 R_1 R_3 + \alpha_4 R_2 R_4)}{R_1 R_3 + R_2 R_4}$$

$$\alpha_s = \frac{R_4}{R_3}(\alpha_4 + \alpha_{ds})$$

Finally,

(answer) $\vec{w}_{ds} = w_{ds}\hat{j}$ (angular velocity of dual shaft)

(answer) $\vec{\alpha}_{ds} = \alpha_{ds}\hat{j}$ (angular acceleration of dual shaft)

$$\vec{w}_2 = \vec{w}_3 = w_{ds}\hat{j} + w_s \cos 30° \hat{i} - w_s \sin 30° \hat{j}$$

(answer) $= w_s \cos 30° \hat{i} + (w_{ds} - w_s \sin 30°)\hat{j}$

(angular velocity of gears 2 and 3)

$$\vec{\alpha}_2 = \vec{\alpha}_3 = \alpha_s \cos 30° \hat{i} + (\alpha_{ds} - \alpha_s \sin 30°)\hat{j} + \vec{w}_{ds} \times \vec{w}_s$$

The w_{ds} rotation vector causes the rotation of the w_s vector. Note that the w_{ds} rotation vector always points along y-axis (constant direction).

$$\vec{\alpha}_2 = \vec{\alpha}_3 = \alpha_s \cos 30° \hat{i} + (\alpha_{ds} - \alpha_s \sin 30°)\hat{j} + w_{ds}\hat{j} \times (w_s \cos 30° \hat{i} - w_s \sin 30° \hat{j})$$

$$= \alpha_s \cos 30° \hat{i} + (\alpha_{ds} - \alpha_s \sin 30°)\hat{j}$$

(answer) $- w_{ds} w_s \cos 30° \hat{k}$ (angular acceleration of gears 2 and 3)