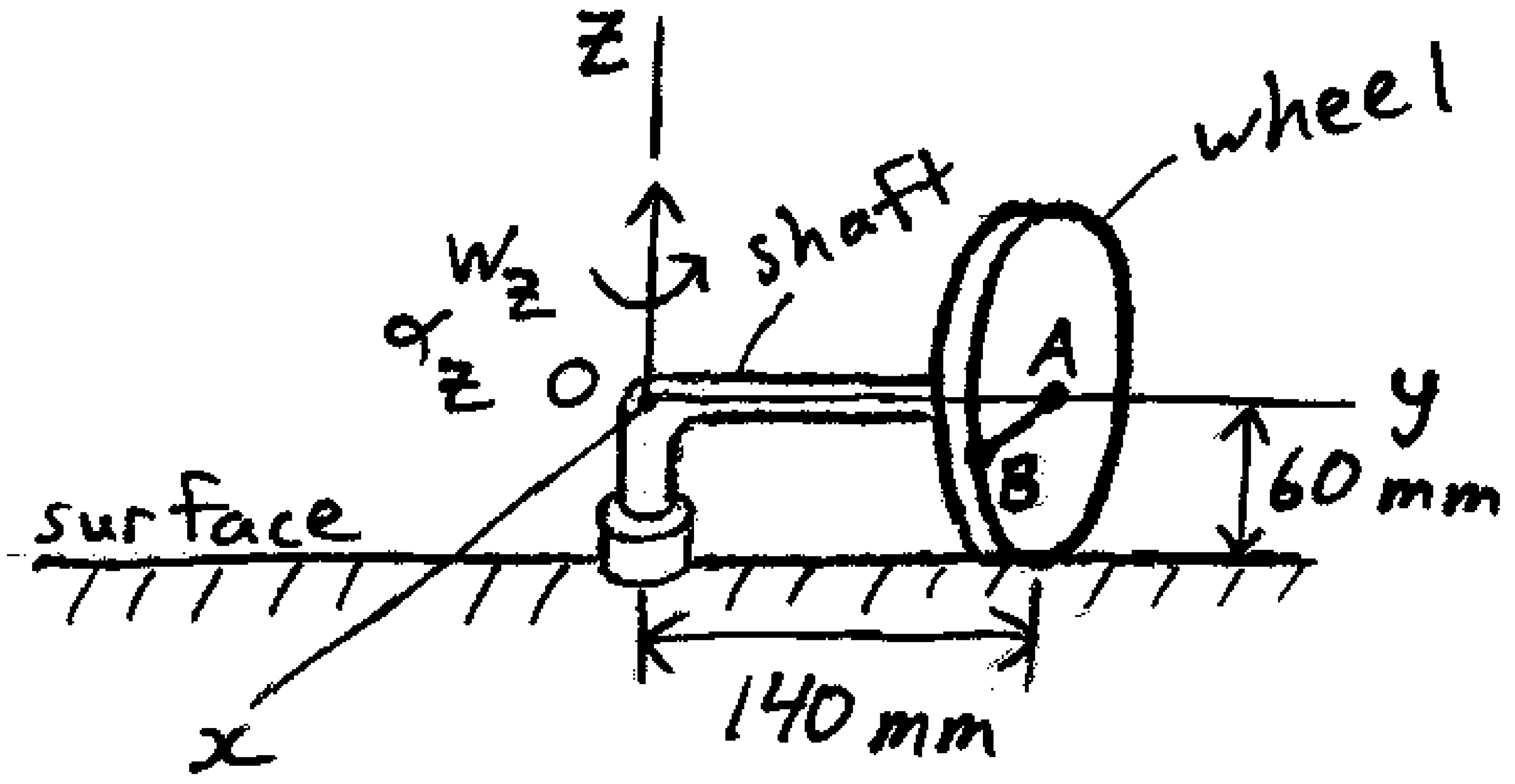
May 11, 2020


This is a 3D general motion problem (engineering mechanics).




A wheel rolls along a level surface without slipping. The wheel rotates on a shaft, and the shaft rotates about point O with an angular velocity of $w_z = 4$ rad/s and an angular acceleration of $\alpha_z = 3$ rad/s$^2$. Determine the velocity and acceleration of point B on the wheel, at the instant shown.

Solution:

The velocity of point A (wheel center) is (4 rad/s)(140 mm) = 560 mm/s, in the negative x-direction. This can be written as:

(This velocity is always tangent to path of A)

$$\vec{V}_A = -560\,\hat{i} \text{ mm/s} \quad (1)$$

(the radius of wheel = 60 mm)

Since the wheel rolls without slipping, the spin component of angular velocity of the wheel is $\frac{V_A}{60\text{ mm}} = \frac{560 \text{ mm/s}}{60 \text{ mm}}$

$= 9.333$ rad/s, in the negative y-direction

This can be written as:

$$\vec{w}_{w,s} = -9.333\,\hat{j} \text{ rad/s}$$

Now, the tangential acceleration of point A (the component of acceleration of point A that is tangent to the path of A) is $(3 \text{ rad/s}^2)(140 \text{ mm}) = 420 \text{ mm/s}^2$, in the negative x-direction

This can be written as:

$$\vec{a}_{A,t} = -420\,\hat{i} \text{ mm/s}^2$$

Since the wheel rolls without slipping, the spin component of angular acceleration of the wheel is $\frac{a_{A,t}}{60 \text{ mm}} = \frac{420 \text{ mm/s}^2}{60 \text{ mm}} = 7 \text{ rad/s}^2$, in the negative y-direction

This can be written as:

$$\vec{\alpha}_{w,s} = -7\,\hat{j} \text{ rad/s}^2$$

The acceleration of point A is the sum of the tangential component plus the normal (centripetal) component:

$$\vec{a}_A = \underbrace{\vec{a}_{A,t}}_{\text{tangential}} + \underbrace{\vec{a}_{A,n}}_{\text{normal}}$$

$$\vec{a}_{A,n} = -\frac{V_A^2}{140 \text{ mm}}\,\hat{j} = -\frac{(560)^2}{140}\,\hat{j} = -2240\,\hat{j} \text{ mm/s}^2$$

Therefore,

$$\vec{a}_A = -420\,\hat{i} - 2240\,\hat{j} \ \ mm/s^2 \quad (2)$$

Next,

$$\vec{w}_w = \vec{w}_{w,s} + \vec{w}_z$$

$$\vec{w}_w = -9.333\,\hat{j} + 4\,\hat{k} \quad (3)$$ (angular velocity of wheel)

Now, determine the angular acceleration of the wheel:

$$\vec{\alpha}_w = \alpha_z\,\hat{k} + \vec{\alpha}_{w,s} + \vec{w}_z \times \vec{w}_{w,s}$$

$$\vec{\alpha}_w = \alpha_z\,\hat{k} + \vec{\alpha}_{w,s} + w_z\,\hat{k} \times (-9.333\,\hat{j})$$

The $w_z$ rotation vector causes the rotation of the $w_{w,s}$ vector. The $w_z$ rotation vector always points along z-axis (constant direction)

Substitute known quantities:

$$\vec{\alpha}_w = 3\,\hat{k} - 7\,\hat{j} + (4)(9.333)\,\hat{i}$$

$$\vec{\alpha}_w = 37.333\,\hat{i} - 7\,\hat{j} + 3\,\hat{k} \quad (4)$$ (angular acceleration of wheel)

Now,

$$\vec{V}_B = \vec{V}_A + \vec{\omega}_w \times \vec{r}_{B/A}$$

$$\vec{r}_{B/A} = 60\hat{i} \quad (5)$$

Substitute known quantities, from (1), (3), and (5):

$$\vec{V}_B = -560\hat{i} + (-9.333\hat{j} + 4\hat{k}) \times (60\hat{i})$$

$$\Rightarrow \vec{V}_B = -560\hat{i} + 240\hat{j} + 560\hat{k} \quad \text{(answer)}$$

mm/s

Lastly,

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_w \times \vec{r}_{B/A} + \vec{\omega}_w \times (\vec{\omega}_w \times \vec{r}_{B/A})$$

Substitute known quantities, from (2), (4), (3), and (5):

$$\vec{a}_B = -420\hat{i} - 2240\hat{j} + (37.333\hat{i} - 7\hat{j} + 3\hat{k}) \times (60\hat{i}) + (-9.333\hat{j} + 4\hat{k}) \times [(-9.333\hat{j} + 4\hat{k}) \times (60\hat{i})]$$

$$\Rightarrow \vec{a}_B = -6606.3\hat{i} - 2060\hat{j} + 420\hat{k} \quad \text{(answer)}$$

$mm/s^2$