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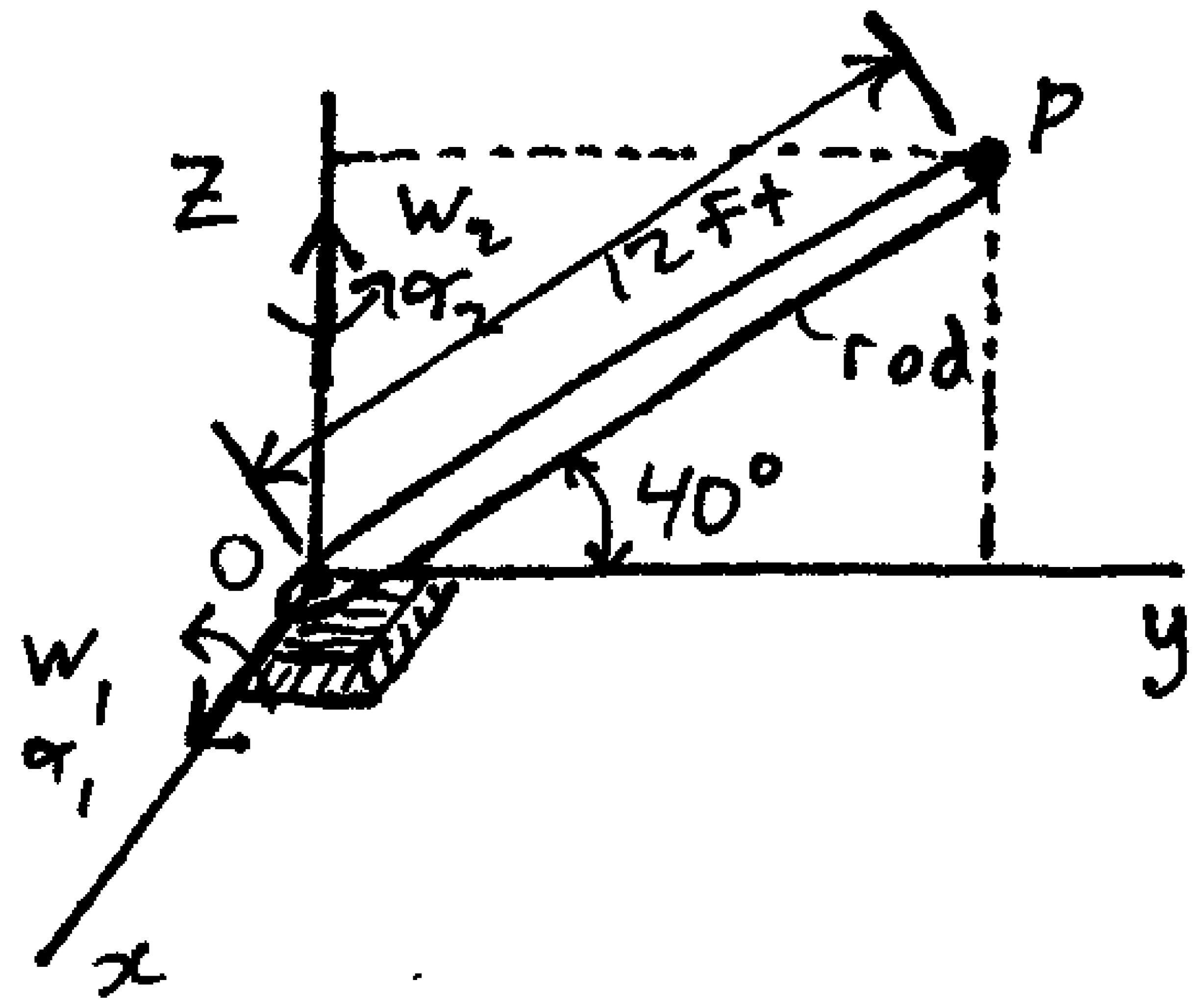
This is a 3D general motion problem (engineering mechanics).



A rod is rotating about the z-axis with an angular velocity $w_2 = 0.25$ rad/s, and an angular acceleration $\alpha_2 = 0.43$ rad/s². At the same time the rod is rotating upward at an angular velocity $w_1 = 0.7$ rad/s, and an angular acceleration $\alpha_1 = 0.35$ rad/s². At the instant shown, calculate the velocity and acceleration of point P at the end of the rod.

Solution:

note: no $\hat{j}$ component

$$\vec{w} = w_1 \hat{i} + w_2 \hat{k}$$ (angular velocity vector of rod)

$$\vec{\alpha} = \alpha_1 \hat{i} + \alpha_2 \hat{k} + \vec{w}_2 \times \vec{w}_1$$ (angular acceleration vector of rod)

The w_2 rotation vector causes the rotation of the w_1 rotation vector. The w_2 rotation vector always points along z-axis (constant direction)

$$\Rightarrow \vec{\alpha} = \alpha_1 \hat{i} + \alpha_2 \hat{k} + w_2 \hat{k} \times w_1 \hat{i}$$

$$\vec{\alpha} = \alpha_1 \hat{i} + \alpha_2 \hat{k} + w_1 w_2 \hat{j}$$

Now, $\vec{V}_P = \vec{V}_0 + \vec{w} \times \vec{r}_{P/O}$

$\vec{V}_0 \to 0$

$$\vec{r}_{P/O} = 12\cos 40° \hat{j} + 12 \sin 40° \hat{k}$$

Substitute known quantities:

$$\vec{V}_P = (0.7\hat{i} + 0.25\hat{k}) \times (12\cos 40° \hat{j} + 12\sin 40° \hat{k})$$

$$\Rightarrow \vec{V}_P = -2.3\hat{i} - 5.4\hat{j} + 6.43\hat{k} \text{ ft/s} \quad \text{(answer)}$$

(algebra not shown)

Next, $\vec{a}_P = \vec{a}_0 + \vec{\alpha} \times \vec{r}_{P/O} + \vec{w} \times (\vec{w} \times \vec{r}_{P/O})$

$\vec{a}_0 \to 0$

Substitute known quantities:

$$\vec{a}_P = (0.35\hat{i} + 0.43\hat{k} + (0.25)(0.7)\hat{j}) \times (12\cos 40° \hat{j} + 12\sin 40° \hat{k}) + (0.7\hat{i} + 0.25\hat{k}) \times [(0.7\hat{i} + 0.25\hat{k}) \times (12\cos 40° \hat{j} + 12 \sin 40° \hat{k})]$$

$$\Rightarrow \vec{a}_P = -1.25\hat{i} - 7.78\hat{j} - 0.56\hat{k} \text{ ft/s}^2 \quad \text{(answer)}$$

(algebra not shown)