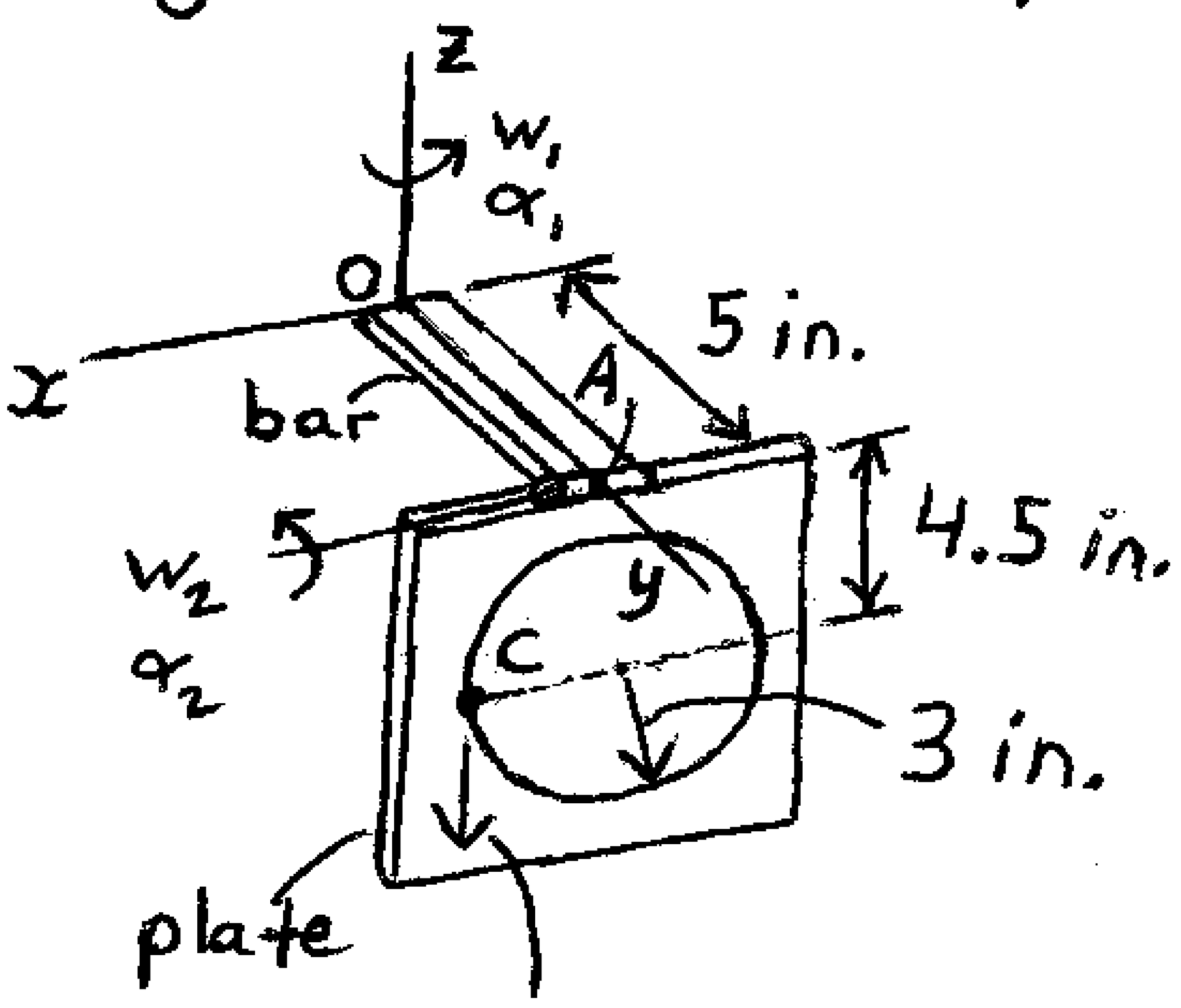
real-world-physics-problems.com
July 20, 2020

This is a 3D general motion problem (engineering mechanics).



$v_{rel} = 6$ in./s
$a_{rel,t} = 7$ in./s^2

A bar is rotating about the z-axis with an angular velocity of $\omega_1 = 2$ rad/s, and an angular acceleration of $\alpha_1 = 3$ rad/s^2, as shown. At the same instant, a plate that is connected to the bar, is rotating relative to the bar at an angular velocity of $\omega_2 = 4$ rad/s and an angular acceleration of $\alpha_2 = 5$ rad/s^2. If a ball C is sliding along the edge of a circular hole cut into the plate, with a velocity of 6 in./s and a tangential acceleration of 7 in./s^2, both measured relative to the plate, what is the velocity and acceleration of the ball C at the instant shown?

Solution:

$\vec{V}_C = \vec{V}_A + \vec{w} \times \vec{r}_{C/A} + (\vec{V}_{C/A})_{rel}$, where $\vec{w}$ is the angular velocity of the plate

$\vec{V}_A = \underset{0}{\vec{V}_o} + \vec{w}_1 \times \vec{r}_{A/O}$

$\vec{r}_{C/A} = 3\hat{i} - 4.5\hat{k}$

$(\vec{V}_{C/A})_{rel} = -6\hat{k}$

$\vec{w}_1 = w_1\hat{k}$ ⇒ (note: if w_1 is shown as a rotation in the opposite direction, then $\vec{w}_1 = -w_1\hat{k}$ → the same is true for all other rotations involving w and $\vec{r}$)

$\vec{r}_{A/O} = 5\hat{j}$

$\vec{w} = w_2\hat{i} + w_1\hat{k}$

Substitute:

$\vec{V}_C = \vec{w}_1 \times \vec{r}_{A/O} + \vec{w} \times \vec{r}_{C/A} + (\vec{V}_{C/A})_{rel}$

Now,

$$\vec{V}_C = (w_1\hat{k}) \times (5\hat{j}) + (w_2\hat{i} + w_1\hat{k}) \times (3\hat{i} - 4.5\hat{k}) - 6\hat{k}$$

$$\vec{V}_C = (2\hat{k}) \times (5\hat{j}) + (4\hat{i} + 2\hat{k}) \times (3\hat{i} - 4.5\hat{k}) - 6\hat{k}$$

$$\vec{V}_C = -10\hat{i} + 24\hat{j} - 6\hat{k} \text{ in./s}$$ (answer)

Next,

$$\vec{a}_C = \vec{a}_A + \vec{\alpha} \times \vec{r}_{C/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{C/A}) + 2\vec{\omega} \times (\vec{v}_{C/A})_{rel} + (\vec{a}_{C/A})_{rel}$$

$$\vec{a}_A = \underbrace{\vec{a}_0}_{0} + \vec{\alpha}_1 \times \vec{r}_{A/0} + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}_{A/0})$$

where $\vec{\alpha}$ is the angular acceleration of the plate

$$(\vec{a}_{C/A})_{rel} = \vec{a}_{rel,t} + \underbrace{\vec{a}_{rel,c}}_{\text{centripetal acceleration component } \left(= -\frac{(v_{rel})^2}{3}\hat{i}\right)}$$

$$\Rightarrow (\vec{a}_{C/A})_{rel} = -7\hat{k} - \frac{(6)^2}{3}\hat{i}$$

$$\Rightarrow (\vec{a}_{C/A})_{rel} = -12\hat{i} - 7\hat{k}$$

$$\vec{\alpha}_1 = \alpha_1 \hat{k}$$

$$\vec{\alpha} = \alpha_2 \hat{i} + \alpha_1 \hat{k} + \vec{\omega}_1 \times \vec{\omega}_2$$

$$\Rightarrow \vec{\alpha} = \alpha_2 \hat{i} + \alpha_1 \hat{k} + (\omega_1 \hat{k}) \times (\omega_2 \hat{i})$$

$$\Rightarrow \vec{\alpha} = \alpha_2 \hat{i} + \omega_1 \omega_2 \hat{j} + \alpha_1 \hat{k}$$

Substitute:

$$\vec{a}_C = \vec{\alpha}_1 \times \vec{r}_{A/O} + \vec{\omega}_1 \times (\omega_1 \times \vec{r}_{A/O}) + \vec{\alpha} \times \vec{r}_{C/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{C/A}) + 2\vec{\omega} \times (\vec{v}_{C/A})_{rel} + (\vec{a}_{C/A})_{rel}$$

$$\vec{a}_C = (3\hat{k}) \times (5\hat{j}) + (2\hat{k}) \times [(2\hat{k}) \times (5\hat{j})] + (5\hat{i} + (2)(4)\hat{j} + 3\hat{k}) \times (3\hat{i} - 4.5\hat{k}) + (4\hat{i} + 2\hat{k}) \times [(4\hat{i} + 2\hat{k}) \times (3\hat{i} - 4.5\hat{k})] + 2(4\hat{i} + 2\hat{k}) \times (-6\hat{k}) - 12\hat{i} - 7\hat{k}$$

$$\vec{a}_C = -111.0\hat{i} + 59.5\hat{j} + 65.0\hat{k} \text{ in./s}^2 \text{ (answer)}$$