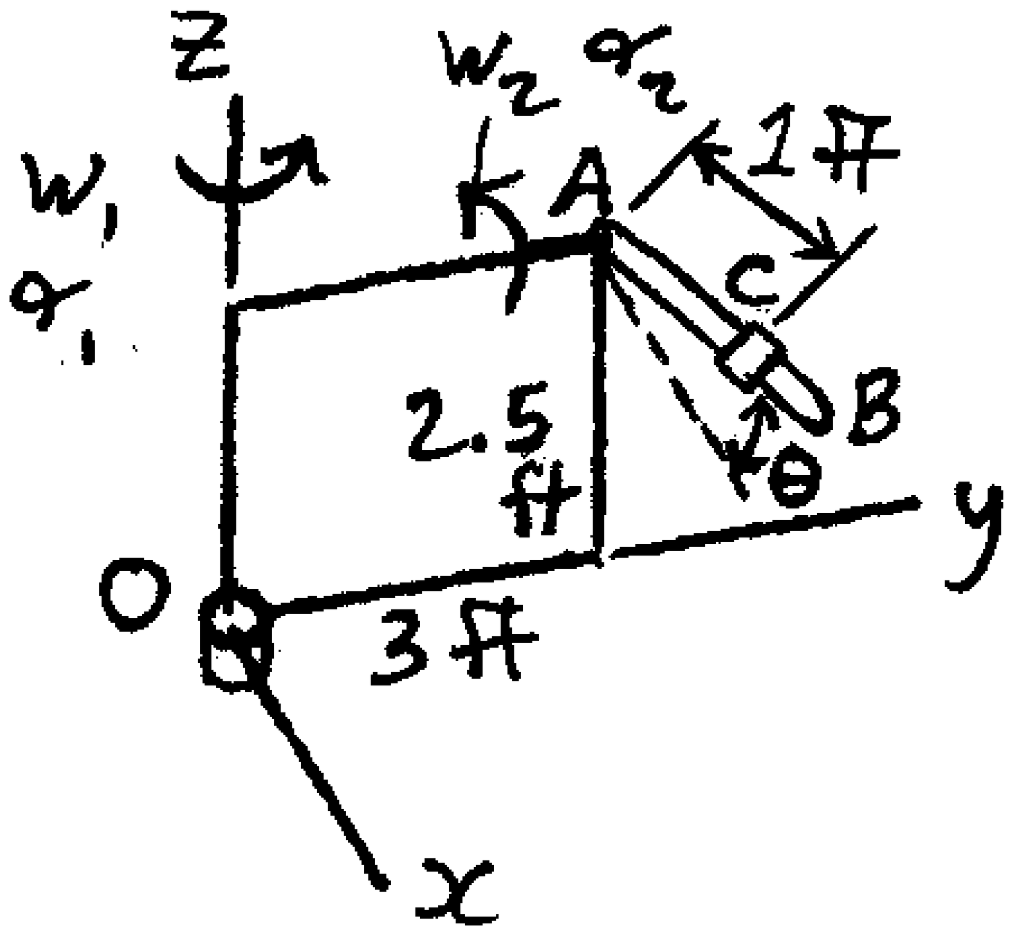
This is a 3D general motion problem (engineering mechanics).



A rectangular plate is rotating about the z-axis with an angular velocity of $w_1 = 0.8$ rad/s, and an angular acceleration of $\alpha_1 = 1.2$ rad/s^2, as shown. At the same instant, a rod AB that is connected to the plate, is rotating relative to the plate at an angular velocity of $w_2 = 2$ rad/s and an angular acceleration of $\alpha_2 = 3$ rad/s^2. If $\theta = 30°$, the plate side lengths are 3 ft and 2.5 ft, the distance between the collar C and point A is 1 ft, and C slides towards A with a velocity of 1.5 ft/s and a deceleration of −2.1 ft/s^2, both measured relative to the rod, what's is the velocity and acceleration of the collar C at the instant shown?

Solution:

$\vec{V}_C = \vec{V}_A + \vec{w} \times \vec{r}_{C/A} + (\vec{V}_{C/A})_{rel}$, where $\vec{w}$ is the angular velocity of the rod AB.

$\vec{V}_A = \vec{V}_o + \vec{w}_1 \times \vec{r}_{A/O}$

$\vec{V}_o \rightarrow 0$

$\vec{r}_{C/A} = 1\cos\theta\,\hat{i} + 1\sin\theta\,\hat{k}$

$(\vec{V}_{C/A})_{rel} = -1.5\cos\theta\,\hat{i} - 1.5\sin\theta\,\hat{k}$

$\vec{w}_1 = w_1\,\hat{k}$

$\vec{r}_{A/O} = 3\,\hat{j} + 2.5\,\hat{k}$

$\vec{w} = -w_2\,\hat{j} + w_1\,\hat{k}$

Substitute:

$\vec{V}_C = \vec{w}_1 \times \vec{r}_{A/O} + \vec{w} \times \vec{r}_{C/A} + (\vec{V}_{C/A})_{rel}$

Now,

$$\vec{V}_C = (w_1\hat{k}) \times (3\hat{j} + 2.5\hat{k}) + (-w_2\hat{j} + w_1\hat{k}) \times (\cos\theta\,\hat{i} + \sin\theta\,\hat{k}) - 1.5\cos\theta\,\hat{i} - 1.5\sin\theta\,\hat{k}$$

$$\vec{V}_C = (0.8\hat{k}) \times (3\hat{j} + 2.5\hat{k}) + (-2\hat{j} + 0.8\hat{k}) \times (\cos 30^\circ\,\hat{i} + \sin 30^\circ\,\hat{k}) - 1.5\cos 30^\circ\,\hat{i} - 1.5\sin 30^\circ\,\hat{k}$$

$\vec{V}_C = -4.70\,\hat{i} + 0.6928\,\hat{j} + 0.9821\,\hat{k}$ (answer) ft/s

Next,

$$\vec{a}_C = \vec{a}_A + \vec{\alpha} \times \vec{r}_{C/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{C/A}) + 2\vec{\omega} \times (\vec{V}_{C/A})_{rel} + (\vec{a}_{C/A})_{rel}$$

$$\vec{a}_A = \vec{a}_o + \vec{\alpha}_1 \times \vec{r}_{A/O} + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}_{A/O})$$

($\vec{a}_o \to 0$)

where $\vec{\alpha}$ is the angular acceleration of the rod AB

$$(\vec{a}_{C/A})_{rel} = 2.1 \cos\theta \, \hat{i} + 2.1 \sin\theta \, \hat{k}$$

$$\vec{\alpha}_1 = \alpha_1 \hat{k}$$

substitute:

$$\vec{\alpha} = -\alpha_2 \hat{j} + \alpha_1 \hat{k} + \omega_1 \omega_2 \hat{i}$$

$$\vec{a}_C = \vec{\alpha}_1 \times \vec{r}_{A/O} + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}_{A/O}) + \vec{\alpha} \times \vec{r}_{C/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{C/A}) + 2\vec{\omega} \times (\vec{V}_{C/A})_{rel} + (\vec{a}_{C/A})_{rel}$$

$$\vec{a}_C = (1.2\hat{k}) \times (3\hat{j} + 2.5\hat{k}) + (0.8\hat{k}) \times [(0.8\hat{k}) \times (3\hat{j} + 2.5\hat{k})]$$
$$+ ((0.8)(2)\hat{i} - 3\hat{j} + 1.2\hat{k}) \times (\cos 30^\circ \hat{i} + \sin 30^\circ \hat{k})$$
$$+ (-2\hat{j} + 0.8\hat{k}) \times [(-2\hat{j} + 0.8\hat{k}) \times (\cos 30^\circ \hat{i} + \sin 30^\circ \hat{k})]$$
$$+ 2(-2\hat{j} + 0.8\hat{k}) \times (-1.5\cos 30^\circ \hat{i} - 1.5 \sin 30^\circ \hat{k})$$
$$+ 2.1 \cos 30^\circ \hat{i} + 2.1 \sin 30^\circ \hat{k}$$

$\vec{a}_c = -4.30\hat{i} - 4.559\hat{j} - 3.548\hat{k}$ ft/s^2 (answer)