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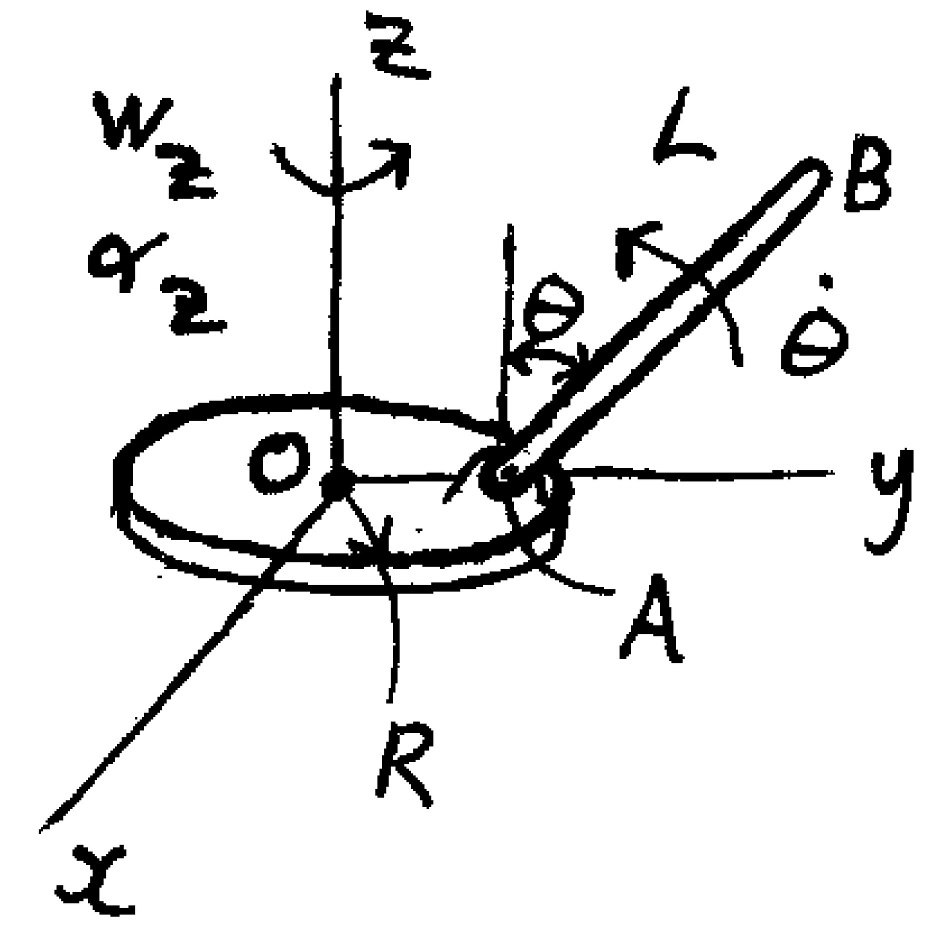
This is a 3D general motion problem (engineering mechanics).



A disk is rotating about the z-axis with an angular velocity of $w_z = 0.85$ rad/s, and an angular acceleration of $\alpha_z = 1.5$ rad/s², as shown. At the same instant, a rod AB that is connected to the disk, is rotating upward at a constant rate of $\dot{\theta} = 0.7$ rad/s. If $\theta = 40°$, the disk radius is $R = 2$ ft, and the rod length is $L = 5$ ft, what is the velocity and acceleration of the rod tip B at the instant shown?

Solution:

$\vec{V}_B = \vec{V}_A + \vec{w} \times \vec{r}_{B/A}$, where $\vec{w}$ is the angular velocity of the rod AB

$\vec{V}_A = \vec{V}_0 + \vec{w}_z \times \vec{r}_{A/O}$ ($\vec{V}_0 \to 0$)

$\vec{r}_{B/A} = L\sin\theta\,\hat{j} + L\cos\theta\,\hat{k}$

$\vec{r}_{A/O} = R\,\hat{j}$

$\vec{w}_z = w_z\hat{k}$

Substitute:

$\vec{V}_B = \vec{w}_z \times \vec{r}_{A/O} + \vec{w} \times \vec{r}_{B/A}$

$\vec{w} = w_z\hat{k} + \dot{\theta}\,\hat{i}$

Now,

$$\vec{V}_B = (w_z\hat{k})\times(R\hat{j}) + (w_z\hat{k} + \dot{\theta}\hat{i})\times(L\sin\theta\hat{j} + L\cos\theta\hat{k})$$

$$\vec{V}_B = (0.85\hat{k})\times(2\hat{j}) + (0.85\hat{k} + 0.7\hat{i})\times(5\sin 40^\circ\hat{j} + 5\cos 40^\circ\hat{k})$$

$$\vec{V}_B = -4.4318\,\hat{i} - 2.6812\,\hat{j} + 2.2498\,\hat{k} \text{ ft/s}$$

(answer)

Next,

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}\times\vec{r}_{B/A} + \vec{w}\times(\vec{w}\times\vec{r}_{B/A})$$

$$\vec{a}_A = \vec{a}_o + \vec{\alpha}_z\times\vec{r}_{A/O} + \underbrace{\vec{w}_z\times(\vec{w}_z\times\vec{r}_{A/O})}_{= -(w_z)^2\,\vec{r}_{A/O}}$$

$\vec{a}_o = 0$

where $\vec{\alpha}$ is the angular acceleration of the rod AB

$$\vec{\alpha}_z = \alpha_z\hat{k}$$

$$\vec{\alpha} = \alpha_z\hat{k} + (w_z)(\dot{\theta})\hat{j}$$

Substitute:

$$\vec{a}_B = \vec{\alpha}_z\times\vec{r}_{A/O} - (w_z)^2\vec{r}_{A/O} + \vec{\alpha}\times\vec{r}_{B/A} + \vec{w}\times(\vec{w}\times\vec{r}_{B/A})$$

Now,

$$\vec{a}_B = (1.5\hat{k}) \times (2\hat{j}) - (0.85)^2(2\hat{j})$$
$$+ (1.5\hat{k} + (0.85)(0.7)\hat{j}) \times (5\sin 40^\circ \hat{j} + 5\cos 40^\circ \hat{k})$$
$$+ (0.85\hat{k} + 0.7\hat{i}) \times [(0.85\hat{k} + 0.7\hat{i}) \times (5\sin 40^\circ \hat{j} + 5\cos 40^\circ \hat{k})]$$

$$\vec{a}_B = -3.2629\hat{i} - 5.3419\hat{j} - 1.8768\hat{k} \text{ ft/s}^2$$
(answer)