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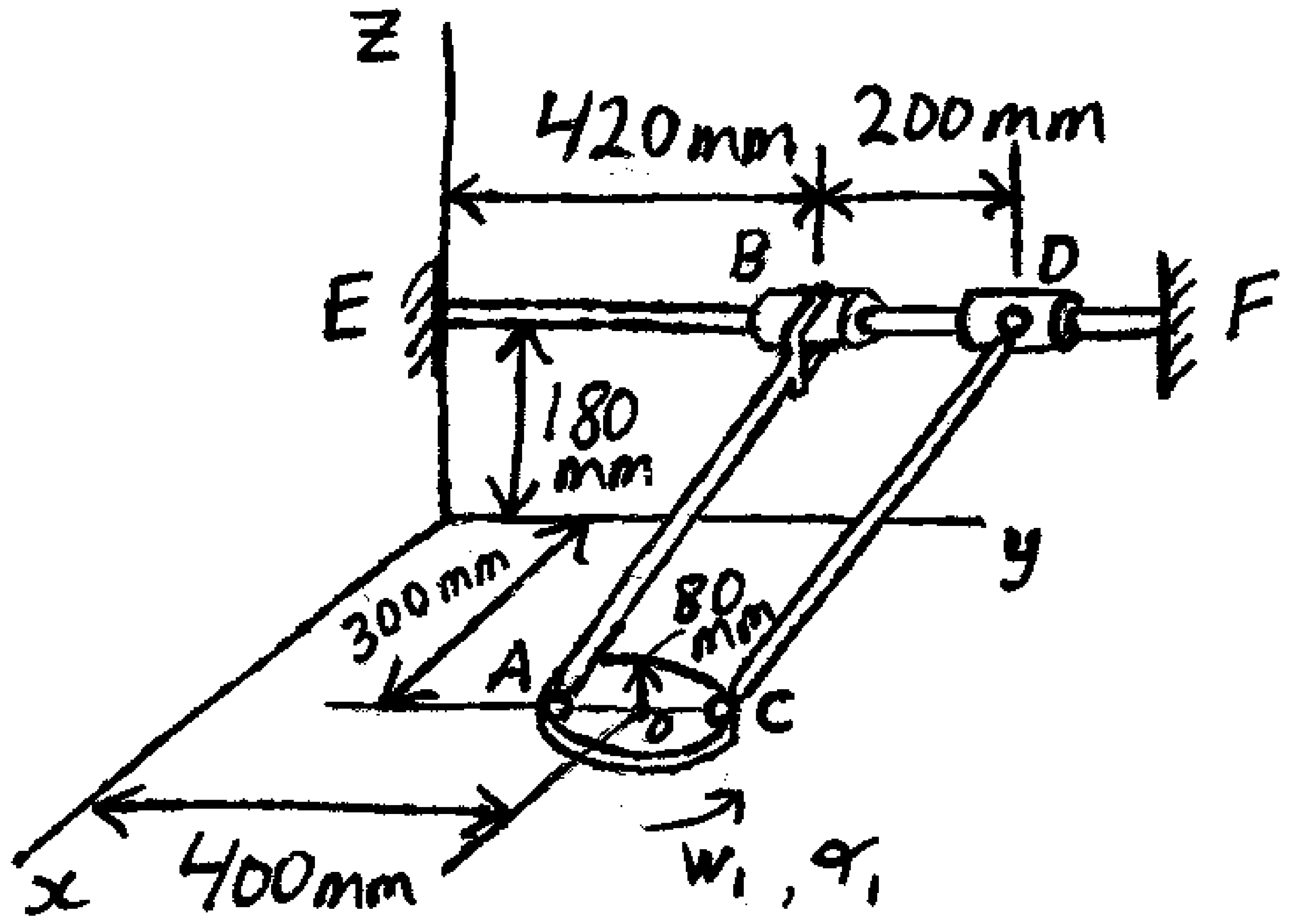
This is a 3D general motion problem (engineering mechanics).



A rod AB and a rod CD are attached to a rotating disk with a ball-and-socket joint at one end. At the other end, rod AB is attached to a clevis, which is attached to a collar that can slide along a horizontal rod EF, in the y-direction. The other end of rod CD is attached to another collar with a ball-and-socket joint, and this collar can also slide along the rod EF. The disk rotates in the xy-plane at an angular velocity of $w_1 = 8$ rad/s, and at an angular acceleration of $\alpha_1 = 7$ rad/s². Determine the velocity and acceleration of both collars, and determine the angular velocity and angular acceleration of rod AB and rod CD, at the instant shown.

Solution:

$\vec{V}_B = \vec{V}_A + \vec{w} \times \vec{r}_{B/A}$

$\vec{V}_A = \vec{V}_o + \vec{w}_1 \times \vec{r}_{A/O}$

$\vec{V}_A = 0 + (8\hat{k}) \times (-80\hat{j})$

$\vec{V}_A = 640\hat{i}$

$\vec{r}_{B/A} = -300\hat{i} + (420 - 400 + 80)\hat{j} + 180\hat{k}$

$\vec{V}_B = V_B\hat{j}$

$\vec{w} = w_x\hat{i} + w_y\hat{j} + w_z\hat{k}$

(angular velocity of rod AB)

Substitute:

$$V_B\hat{j} = 640\hat{i} + (w_x\hat{i} + w_y\hat{j} + w_z\hat{k}) \times (-300\hat{i} + 100\hat{j} + 180\hat{k})$$

Expand this out and equate the respective $\hat{i}, \hat{j}, \hat{k}$ terms. This results in 3 equations:

$$180 w_y - 100 w_z = -640 \quad (1)$$

$$-180 w_x - 300 w_z = V_B \quad (2)$$

$$100 w_x + 300 w_y = 0 \quad (3)$$

These 3 equations contain 4 unknowns. So we need one more equation.

Let $\vec{n}$ be a vector pointing along the axis of the clevis pins. Let $\vec{p}$ be a vector pointing along the axis of the rod EF. Then,

$\vec{p} = \hat{j}$ and $\vec{n} = \vec{r}_{A/B} \times \vec{r}_{A/E}$, $\vec{r}_{A/E} = 300\hat{i} + 320\hat{j} - 180\hat{k}$

$$\vec{n} = (300\hat{i} - 100\hat{j} - 180\hat{k}) \times (300\hat{i} + 320\hat{j} - 180\hat{k})$$

$$\vec{n} = 75600\hat{i} + 126000\hat{k}$$

Set $\vec{u} = \vec{p} \times \vec{n}$

$\vec{u} = \hat{j} \times (75600\hat{i} + 126000\hat{k})$

$\vec{u} = 126000\hat{i} - 75600\hat{k}$

Next, $\vec{w} \cdot \vec{u} = 0$ (due to the constraint of the clevis).

Substitute:

$$126000 w_x - 75600 w_z = 0 \quad (4)$$

Combine equations (1)-(4) and solve for the 4 unknowns:

$$w_x = 2.8235 \text{ rad/s}$$

$$w_y = -0.94118 \text{ rad/s}$$

$$w_z = 4.7059 \text{ rad/s}$$

$$V_B = -1920 \text{ mm/s}$$

(answer)

The angular velocity of rod AB is: $\vec{w} = 2.8235\hat{i} - 0.94118\hat{j} + 4.7059\hat{k}$ rad/s

The velocity of collar B is 1920 mm/s $\leftarrow$

$\vec{V}_B = -1920\hat{j}$ mm/s (answer)

Next, $\vec{\alpha} = \alpha_x \hat{i} + \alpha_y \hat{j} + \alpha_z \hat{k}$ (angular acceleration of rod AB)

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$$

$$\vec{a}_A = \vec{a}_o + \vec{\alpha}_1 \times \vec{r}_{A/O} + \underbrace{\vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}_{A/O})}_{= -(\omega_1)^2 \vec{r}_{A/O}}$$

$$\vec{a}_A = 0 + (7\hat{k}) \times (-80\hat{j}) - (8)^2(-80\hat{j})$$

$$\vec{a}_A = 560\hat{i} + 5120\hat{j}$$

Substitute:

$$\vec{a}_B = 560\hat{i} + 5120\hat{j} + (\alpha_x \hat{i} + \alpha_y \hat{j} + \alpha_z \hat{k}) \times (-300\hat{i} + 100\hat{j} + 180\hat{k})$$
$$+ (2.8235\hat{i} - 0.94118\hat{j} + 4.7059\hat{k})$$
$$\times [(2.8235\hat{i} - 0.94118\hat{j} + 4.7059\hat{k}) \times (-300\hat{i} + 100\hat{j} + 180\hat{k})]$$

$$\vec{a}_B = 9595.33176\hat{i} + 2108.224\hat{j} - 6023.4775\hat{k}$$
$$+ (180\alpha_y - 100\alpha_z)\hat{i} - (180\alpha_x + 300\alpha_z)\hat{j}$$
$$+ (100\alpha_x + 300\alpha_y)\hat{k}$$

Now, $\vec{a}_B = a_B \hat{j}$

Expand this out and equate the respective $\hat{i}, \hat{j}, \hat{k}$ terms. This results in 3 equations:

$180\alpha_y - 100\alpha_z + 9595.33176 = 0 \quad (5)$

$-180\alpha_x - 300\alpha_z + 2108.224 = a_B \quad (6)$

$100\alpha_x + 300\alpha_y - 6023.4775 = 0 \quad (7)$

Next, $\vec{\alpha}\cdot\frac{\vec{u}}{|\vec{u}|} = w_p w_n$ (due to the constraint of the clevis)

where $w_p = \vec{w}\cdot\frac{\vec{p}}{|\vec{p}|}$, $w_n = \vec{w}\cdot\frac{\vec{n}}{|\vec{n}|}$

(precession about rod axis EF)

(nutation about clevis pin axis)

$w_p = -0.94118$ rad/s

$w_n = 5.488$ rad/s

$|\vec{u}| = 146939.9878$

Substitute:

$$\frac{126000\alpha_x - 75600\alpha_z}{146939.9878} = (-0.94118)(5.488) \quad (8)$$

Combine equations (5)-(8) and solve for the 4 unknowns:

$\alpha_x = 53.8477$ rad/s²

$\alpha_y = 2.129$ rad/s²

$\alpha_z = 99.7855$ rad/s² (ans.)

$a_B = -37520.031$ mm/s²

The angular acceleration of rod AB is:

$\vec{\alpha} = 53.8477\hat{i} + 2.129\hat{j} + 99.7855\hat{k}$ rad/s²

The acceleration of collar B is 37520.031 mm/s²

~~$\vec{a}_B = -37520.031\hat{j}$ rad/s²~~

Now, analyze rod CD:

$\vec{V}_D = \vec{V}_C + \vec{w} \times \vec{r}_{D/C}$

$\vec{V}_C = \vec{V}_O + \vec{w}_1 \times \vec{r}_{C/O}$

$\vec{V}_C = 0 + (8\hat{k}) \times (80\hat{j})$

$\vec{V}_C = -640\hat{i}$

$\vec{r}_{D/C} = -300\hat{i} + (420 + 200 - 400 - 80)\hat{j} + 180\hat{k}$

$\vec{V}_D = V_D\hat{j}$

$\vec{w} = w_x\hat{i} + w_y\hat{j} + w_z\hat{k}$

(angular velocity of rod CD)

Substitute:

$$V_D\hat{j} = -640\hat{i} + (w_x\hat{i} + w_y\hat{j} + w_z\hat{k}) \times (-300\hat{i} + 140\hat{j} + 180\hat{k})$$

Expand this out and equate the respective $\hat{i}, \hat{j}, \hat{k}$ terms. This results in 3 equations:

$$180w_y - 140w_z - 640 = 0 \quad (9)$$

$$-180w_x - 300w_z = V_D \quad (10)$$

$$140w_x + 300w_y = 0 \quad (11)$$

These 3 equations contain 4 unknowns. So we need one more equation.

Use $\vec{w} \cdot \vec{r}_{D/C} = 0$

substitute:

$$(w_x \hat{i} + w_y \hat{j} + w_z \hat{k}) \cdot (-300\hat{i} + 140\hat{j} + 180\hat{k}) \qquad (12)$$

$$\Rightarrow -300 w_x + 140 w_y + 180 w_z = 0$$

Combine (9)–(12) and solve for the 4 unknowns:

$$w_x = -1.7384 \text{ rad/s}$$

$$w_y = 0.8113 \text{ rad/s}$$

$$w_z = -3.5284 \text{ rad/s}$$

$$V_D = 1371.4286 \text{ mm/s}$$

The angular velocity of rod CD is: (answer)

$$\vec{w} = -1.7384\hat{i} + 0.8113\hat{j} - 3.5284\hat{k} \text{ rad/s}$$

The velocity of collar D is: 1371.4286 mm/s $\rightarrow$

$$\vec{V}_D = 1371.4286\hat{j} \text{ mm/s} \quad \text{(answer)}$$

$\vec{\alpha} = \alpha_x \hat{i} + \alpha_y \hat{j} + \alpha_z \hat{k}$ (angular acceleration of rod CD)

Next,

$\vec{a}_D = \vec{a}_C + \vec{\alpha} \times \vec{r}_{D/C} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{D/C})$

$\vec{a}_C = \vec{a}_o + \vec{\alpha}_1 \times \vec{r}_{C/O} + \underbrace{\vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}_{C/O})}_{= -(\omega_1)^2 \vec{r}_{C/O}}$

$\vec{a}_C = 0 + (7\hat{k}) \times (80\hat{j}) - (8)^2 (80\hat{j})$

$\vec{a}_C = -560\hat{i} - 5120\hat{j}$

substitute:

$$\vec{a}_D = -560\hat{i} - 5120\hat{j} + (\alpha_x \hat{i} + \alpha_y \hat{j} + \alpha_z \hat{k}) \times (-300\hat{i} + 140\hat{j} + 180\hat{k})$$
$$+ (-1.7384\hat{i} + 0.8113\hat{j} - 3.5284\hat{k}) \times [(-1.7384\hat{i} + 0.8113\hat{j} - 3.5284\hat{k}) \times (-300\hat{i} + 140\hat{j} + 180\hat{k})]$$

$$\vec{a}_D = -560\hat{i} - 5120\hat{j} + (180\alpha_y - 140\alpha_z)\hat{i} + (-180\alpha_x - 300\alpha_z)\hat{j} + (140\alpha_x + 300\alpha_y)\hat{k}$$
$$+ 4838.972\hat{i} - 2258.1869\hat{j} - 2903.3375\hat{k}$$

Now, $\vec{a}_D = a_D \hat{j}$

Expand this out and equate the respective $\hat{i}, \hat{j}, \hat{k}$ terms. This results in 3 equations:

$$180\alpha_y - 140\alpha_z - 560 + 4838.972 = 0 \quad (13)$$

$$-180\alpha_x - 300\alpha_z - 5120 - 2258.1869 = a_D \quad (14)$$

$$140\alpha_x + 300\alpha_y - 2903.3375 = 0 \quad (15)$$

One more equation is needed:

$$\vec{\alpha} \cdot \vec{r}_{D/C} = 0$$

substitute:

$$(\alpha_x \hat{i} + \alpha_y \hat{j} + \alpha_z \hat{k}) \cdot (-300\hat{i} + 140\hat{j} + 180\hat{k}) = 0$$

$$\Rightarrow -300\alpha_x + 140\alpha_y + 180\alpha_z = 0 \quad (16)$$

combine (13)-(16) and solve for the 4 unknowns:

$\alpha_x = 19.2172$ rad/s^2

$\alpha_y = 0.7098$ rad/s^2

$\alpha_z = 31.4766$ rad/s^2

$a_D = -20280.2751$ mm/s^2

(answer) the angular acceleration of rod CD is:

$$\vec{\alpha} = 19.2172\hat{i} + 0.7098\hat{j} + 31.4766\hat{k} \text{ rad/s}^2$$

The acceleration of collar D is:

$$\vec{a}_D = -20280.2751\hat{j} \text{ mm/s}^2$$

(answer)

20280.2751 mm/s^2 $\leftarrow$