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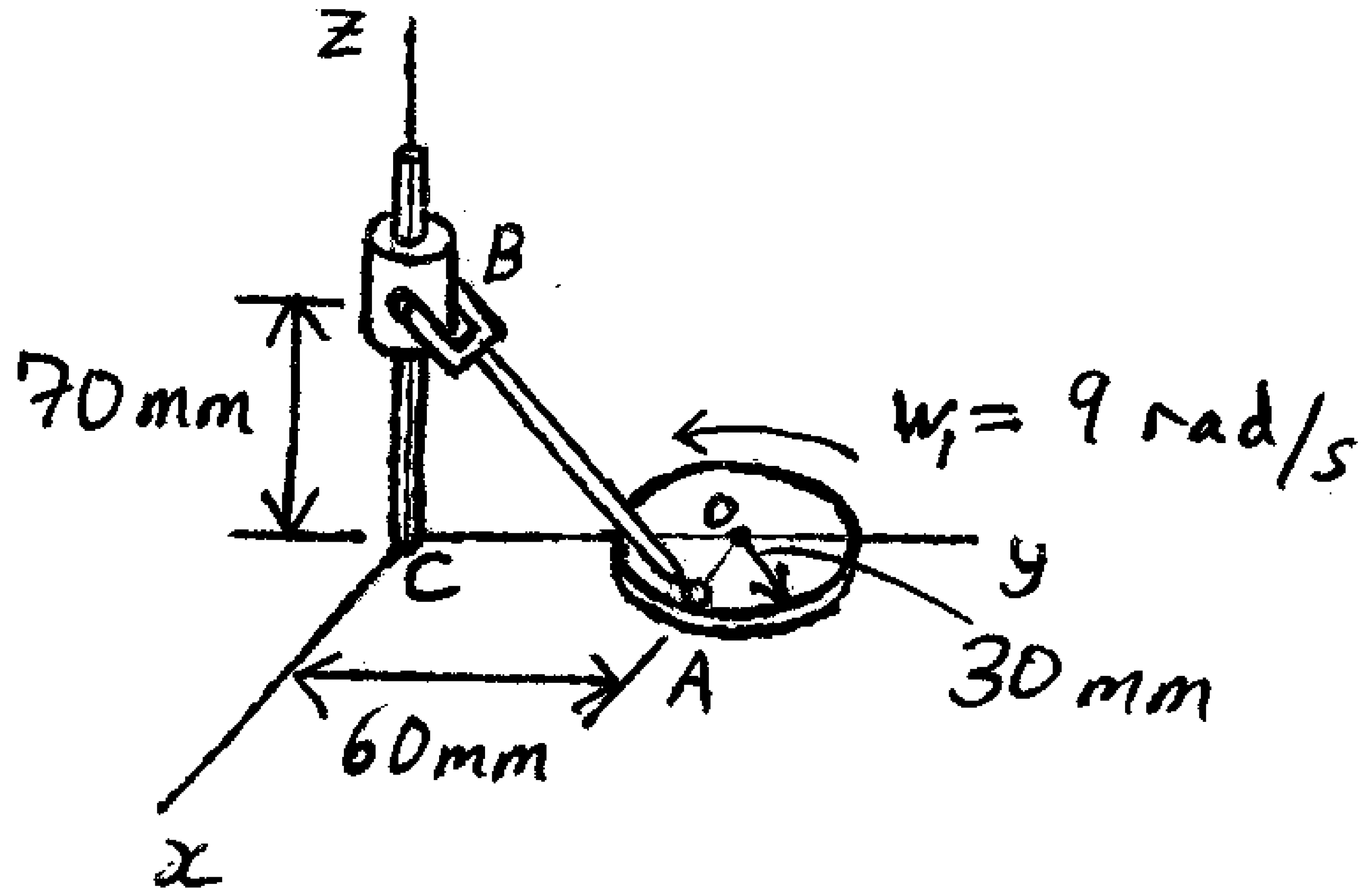
This is a 3D general motion problem (engineering mechanics).



A rod AB is attached to a rotating disk at one end A with a ball-and-socket joint, and the other end B is attached to a clevis, which is attached to a collar that can slide along a vertical rod in the z-direction. The disk rotates in the xy-plane at a constant rate of $w_1 = 9$ rad/s. Determine the angular velocity of the rod, and the velocity of the collar, at the instant shown.

Solution:

$$\vec{V}_B = \vec{V}_A + \vec{w} \times \vec{r}_{B/A}$$

$$\vec{r}_{B/A} = -30\hat{i} - 60\hat{j} + 70\hat{k} \text{ mm}$$

$$\vec{V}_B = V_B \hat{k}$$

$$\vec{w} = w_x \hat{i} + w_y \hat{j} + w_z \hat{k}$$

(angular velocity of rod)

$$\vec{V}_A = \vec{V}_0 + \vec{w}_1 \times \vec{r}_{A/0}$$

$$\vec{V}_A = 0 + 9\hat{k} \times 30\hat{i}$$

$$\vec{V}_A = 270\hat{j} \text{ mm/s}$$

substitute:

$$v_B \hat{k} = 270\hat{j} + (w_x \hat{i} + w_y \hat{j} + w_z \hat{k}) \times (-30\hat{i} - 60\hat{j} + 70\hat{k})$$

Expand this out and equate the respective $\hat{i}, \hat{j}, \hat{k}$ terms. This results in 3 equations:

$$70 w_y + 60 w_z = 0 \quad (1)$$

$$70 w_x + 30 w_z = 270 \quad (2)$$

$$-60 w_x + 30 w_y = v_B \quad (3)$$

These 3 equations contain 4 unknowns. So we need one more equation.

Let $\vec{n}$ be a vector pointing along the axis of the clevis pins. Let $\vec{p}$ be a vector pointing along the axis of the vertical rod. Then,

$$\vec{p} = \hat{k}$$

and $\vec{n} = \vec{r}_{A/B} \times \vec{r}_{A/C}$, $\vec{r}_{A/C} = 30\hat{i} + 60\hat{j}$

$$\vec{n} = (30\hat{i} + 60\hat{j} - 70\hat{k}) \times (30\hat{i} + 60\hat{j})$$

$$\vec{n} = 4200\hat{i} - 2100\hat{j}$$

Set $\vec{u} = \vec{p} \times \vec{n}$

$$\vec{u} = \hat{k} \times (4200\hat{i} - 2100\hat{j})$$

$$\vec{u} = 2100\hat{i} + 4200\hat{j}$$

Next,

$\vec{w} \cdot \vec{u} = 0$ (due to the constraint of the clevis).

So, $\vec{w} \cdot \vec{u} = 0$

$\Rightarrow (w_x \hat{i} + w_y \hat{j} + w_z \hat{k}) \cdot (2100 \hat{i} + 4200 \hat{j}) = 0$

$\Rightarrow 2100 w_x + 4200 w_y = 0$ (4)

Combine equations (1) - (4) and solve for the 4 unknowns:

$w_x = 3.086$ rad/s

$w_y = -1.543$ rad/s

$w_z = 1.8$ rad/s

$V_B = -231.429$ mm/s

The angular velocity of the rod is: $\vec{w} = 3.086 \hat{i} - 1.543 \hat{j} + 1.8 \hat{k}$ rad/s (answer)

The velocity of the collar B is:

231.429 mm/s ↓

$\vec{V}_B = -231.429 \hat{k}$ mm/s (answer)