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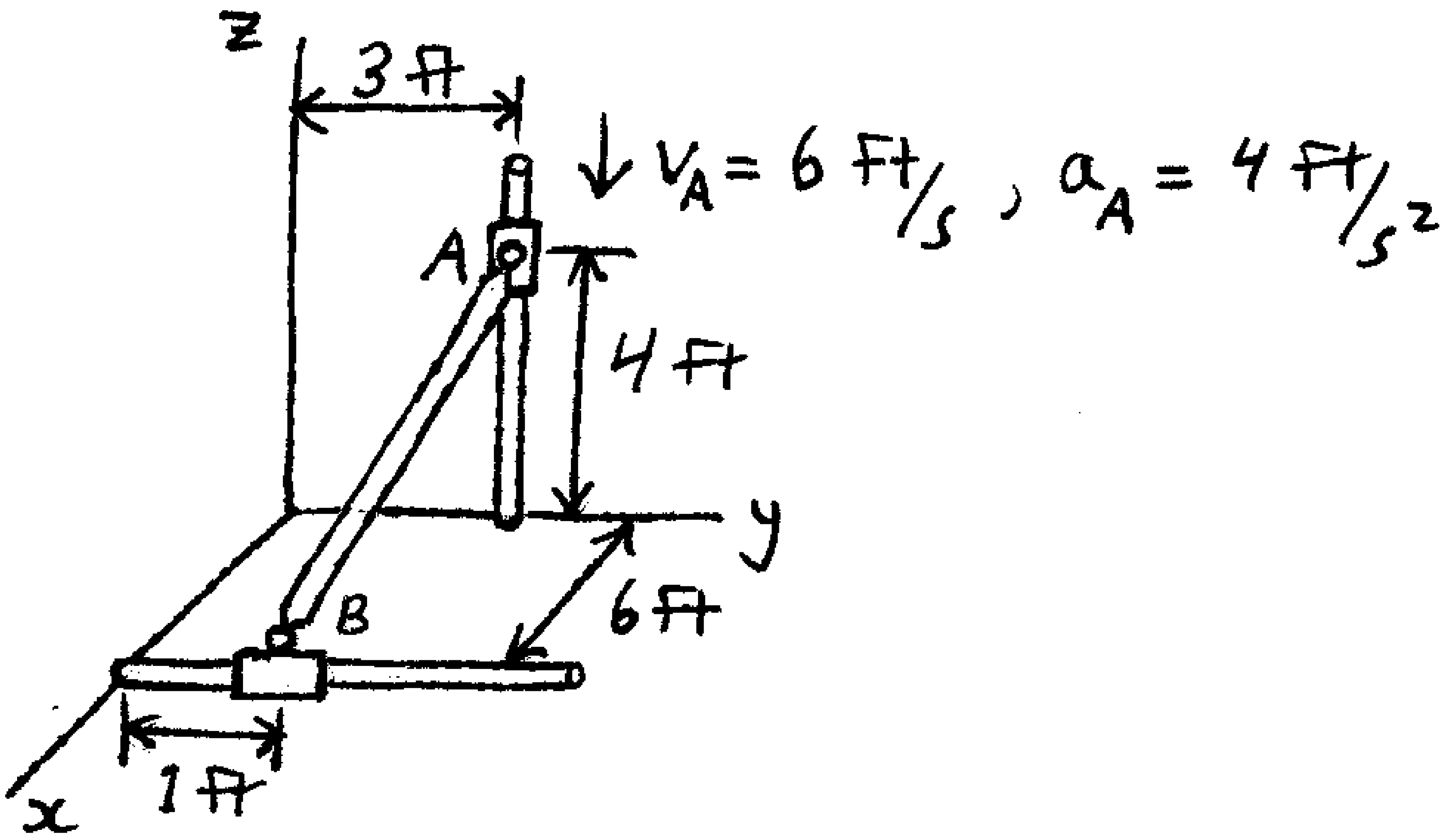
This is a 3D general motion problem (engineering mechanics).



The rod AB has ball-and-socket joints at both ends which are attached to smooth collars A and B. Determine the angular velocity of the rod, and the velocity and acceleration of collar B at the instant shown.

Solution:

$\vec{v}_B = \vec{v}_A + \vec{w} \times \vec{r}_{B/A}$ $\qquad \vec{v}_A = -6\hat{k}$ ft/s

$\vec{r}_{B/A} = 6\hat{i} - 2\hat{j} - 4\hat{k}$ ft

$\vec{v}_B = -v_B\hat{j}$

$\vec{w} = w_x\hat{i} + w_y\hat{j} + w_z\hat{k}$

Substitute:

$$-v_B\hat{j} = -6\hat{k} + (w_x\hat{i} + w_y\hat{j} + w_z\hat{k}) \times (6\hat{i} - 2\hat{j} - 4\hat{k})$$

Expand this out and equate the respective $\hat{i}, \hat{j}, \hat{k}$ terms. This results in 3 equations:

$$-4w_y + 2w_z = 0 \quad (1)$$

$$-4w_x - 6w_z = v_B \quad (2)$$

$$-2w_x - 6w_y = 6 \quad (3)$$

These 3 equations contain 4 unknowns. So we need one more equation.

The rod has ball-and-socket joints at both ends which are attached to smooth collars, with low friction. This means that relative motion can occur between the rod and the collars. However, because of the low friction assumption, there is a negligible moment (torque) to rotate the rod about its (lengthwise) axis. This in turn means that the rod has no angular acceleration component, and consequently, no angular velocity component about its lengthwise axis.

Therefore, $\underbrace{\vec{w} \cdot \vec{r}_{B/A}}_{\text{dot product}} = 0$

Substitute:

$$(w_x\hat{i} + w_y\hat{j} + w_z\hat{k})\cdot(6\hat{i} - 2\hat{j} - 4\hat{k}) = 0$$

$$\Rightarrow 6w_x - 2w_y - 4w_z = 0 \quad (4)$$

Combine (1)–(4) and solve for the 4 unknowns:

$$w_x = -1.071 \text{ rad/s}$$

$$w_y = -0.643 \text{ rad/s}$$

$$w_z = -1.286 \text{ rad/s}$$

$$V_B = 12 \text{ ft/s}$$

The angular velocity of the rod is: $\vec{w} = -1.071\hat{i} - 0.643\hat{j} - 1.286\hat{k}$ rad/s

The velocity of collar B is:

$$V_B = 12 \text{ ft/s} \leftarrow$$

$$\vec{V}_B = -12\hat{j} \text{ ft/s} \quad \text{(answer)}$$

Next,

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}\times\vec{r}_{B/A} + \vec{w}\times(\vec{w}\times\vec{r}_{B/A})$$

$$\vec{a}_A = -4\hat{k} \text{ ft/s}^2$$

$$\vec{\alpha} = \alpha_x\hat{i} + \alpha_y\hat{j} + \alpha_z\hat{k}$$

$$\vec{a}_B = -a_B\hat{j}$$

substitute:

$$-a_B\hat{j} = -4\hat{k} + (\alpha_x\hat{i} + \alpha_y\hat{j} + \alpha_z\hat{k}) \times (6\hat{i} - 2\hat{j} - 4\hat{k})$$
$$+ (-1.071\hat{i} - 0.643\hat{j} - 1.286\hat{k})$$
$$\times [(-1.071\hat{i} - 0.643\hat{j} - 1.286\hat{k}) \times (6\hat{i} - 2\hat{j} - 4\hat{k})]$$

$$\Rightarrow -a_B\hat{j} = -4\hat{k} + (-4\alpha_y + 2\alpha_z)\hat{i} - (-4\alpha_x - 6\alpha_z)\hat{j} + (-2\alpha_x - 6\alpha_y)\hat{k}$$
$$- 19.29\hat{i} + 6.426\hat{j} + 12.852\hat{k}$$

Now, equate the respective $\hat{i}, \hat{j}, \hat{k}$ terms. This results in 3 equations:

$$-4\alpha_y + 2\alpha_z = 19.29 \quad (5)$$
$$-4\alpha_x - 6\alpha_z - 6.426 = a_B \quad (6)$$
$$-2\alpha_x - 6\alpha_y = -8.852 \quad (7)$$

One more equation is needed:

$$\vec{\alpha} \cdot \vec{r}_{B/A} = 0$$

Substitute:

$$(\alpha_x \hat{i} + \alpha_y \hat{j} + \alpha_z \hat{k}) \cdot (6\hat{i} - 2\hat{j} - 4\hat{k}) = 0$$

$$\Rightarrow \quad 6\alpha_x - 2\alpha_y - 4\alpha_z = 0 \quad (8)$$

Combine (5) - (8) and solve for the 4 unknowns:

$$\alpha_x = 5.714 \ \text{rad}/s^2$$

$$\alpha_y = -0.429 \ \text{rad}/s^2$$

$$\alpha_z = 8.786 \ \text{rad}/s^2$$

$$a_B = -82 \ \text{ft}/s^2$$

The acceleration of collar B is $82 \ \text{ft}/s^2 \rightarrow$

$$\vec{a}_B = 82 \hat{j} \ \text{ft}/s^2$$

(answer)