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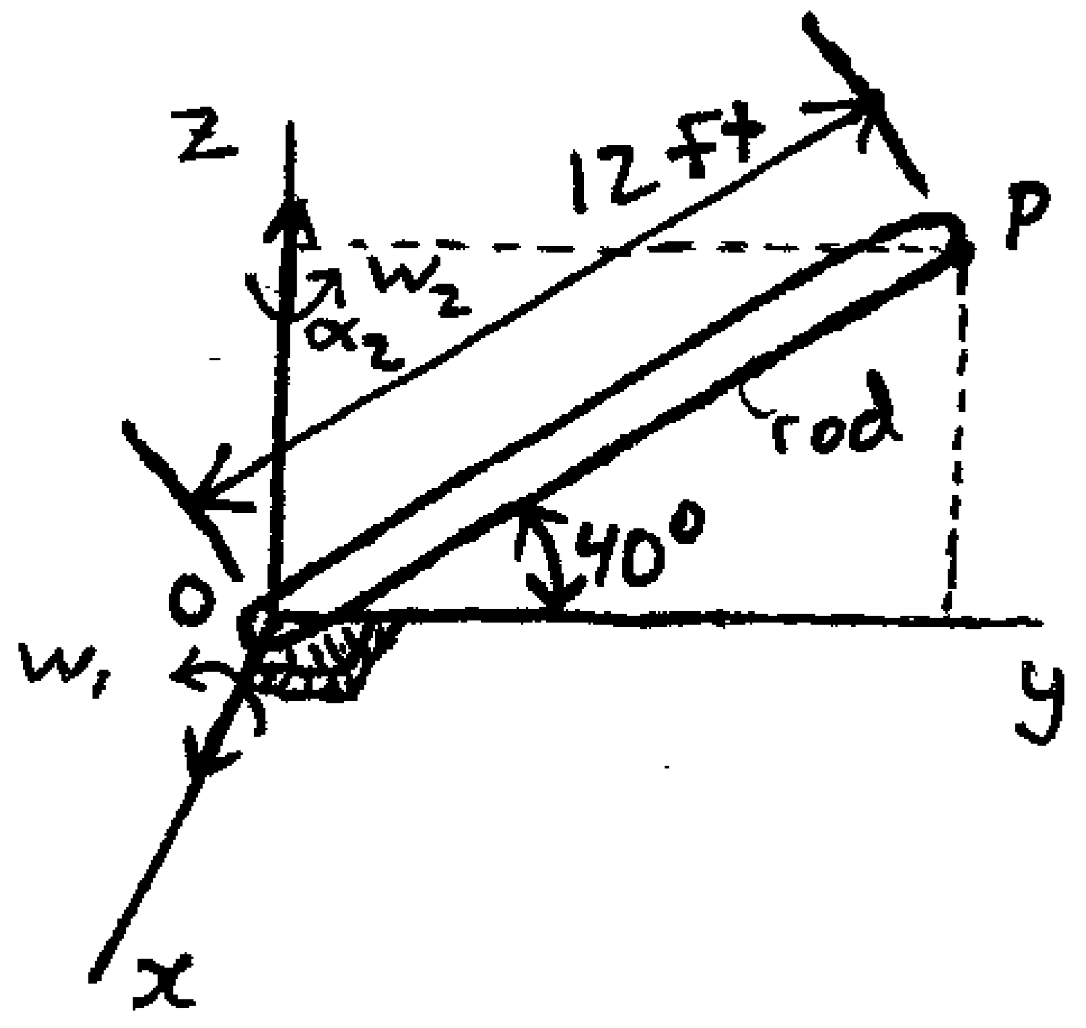
This is a 3D general motion problem (engineering mechanics).



A rod is rotating about the z-axis with an angular velocity $w_2 = 0.25$ rad/s, and an angular acceleration $\alpha_2 = 0.52$ rad/s². At the same time the rod is rotating upward at an angular velocity $w_1 = 0.7$ rad/s, which is constant. At the instant shown, calculate the velocity and acceleration of point P at the end of the rod.

<u>Solution:</u>

note: no $\hat{j}$ component

$$\vec{w} = w_1\hat{i} + w_2\hat{k}$$ (angular velocity vector of rod)

$$\vec{\alpha} = \alpha_2\hat{k} + \vec{w}_2 \times \vec{w}_1$$ (angular acceleration vector of rod)

The w_2 rotation vector causes the rotation of the w_1 rotation vector. The w_2 rotation vector always points along z-axis (constant direction)

$$\Rightarrow \vec{\alpha} = \alpha_2\hat{k} + w_2\hat{k} \times w_1\hat{i} \Rightarrow \vec{\alpha} = \alpha_2\hat{k} + w_1 w_2\hat{j}$$

note: no $\hat{i}$ component

Now, $\vec{v}_p = \vec{v}_o + \vec{w} \times \vec{r}_{p/o}$

$\vec{v}_o \to 0$

$\vec{r}_{p/o} = 12\cos 40^\circ \hat{j} + 12 \sin 40^\circ \hat{k}$

Substitute known quantities:

$$\vec{v}_p = (w_1 \hat{i} + w_2 \hat{k}) \times (12 \cos 40^\circ \hat{j} + 12 \sin 40^\circ \hat{k})$$

$$\vec{v}_p = (w_1 \hat{i} + w_2 \hat{k}) \times (12 \cos 40^\circ \hat{j}) + (w_1 \hat{i} + w_2 \hat{k}) \times (12 \sin 40^\circ \hat{k})$$

For $w_1 = 0.7$ rad/s and $w_2 = 0.25$ rad/s:

$\Rightarrow \vec{v}_p = -2.3\hat{i} - 5.4\hat{j} + 6.43\hat{k}$ ft/s (answer) (algebra not shown)

Next, $\vec{a}_p = \vec{a}_o + \vec{\alpha} \times \vec{r}_{p/o} + \vec{w} \times (\vec{w} \times \vec{r}_{p/o})$

$\vec{a}_o \to 0$

Substitute known quantities:

$$\vec{a}_p = (\alpha_2 \hat{k} + w_1 w_2 \hat{j}) \times (12\cos 40^\circ \hat{j} + 12 \sin 40^\circ \hat{k}) + (w_1 \hat{i} + w_2 \hat{k}) \times [(w_1 \hat{i} + w_2 \hat{k}) \times (12 \cos 40^\circ \hat{j} + 12 \sin 40^\circ \hat{k})]$$

For $w_1 = 0.7$ rad/s, $w_2 = 0.25$ rad/s, and $\alpha_2 = 0.52$ rad/s²:

$\Rightarrow \vec{a}_p = -2.08\hat{i} - 5.08\hat{j} - 3.78\hat{k}$ ft/s² (answer)
(algebra not shown)