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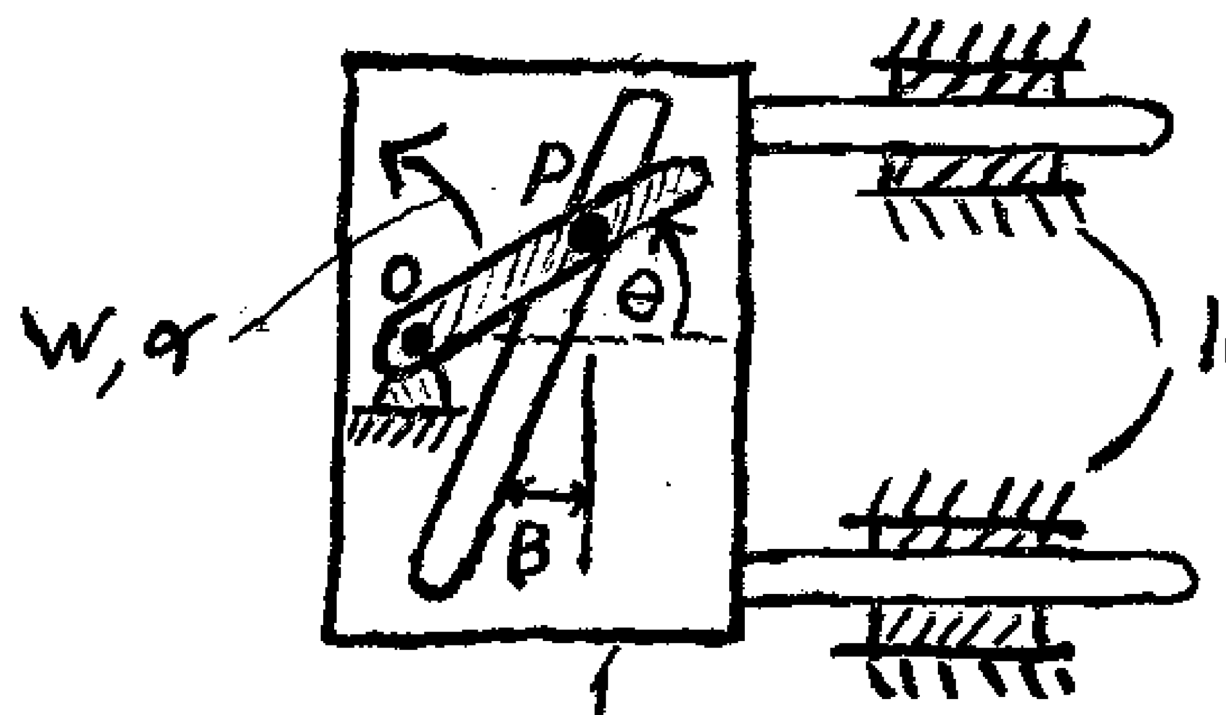
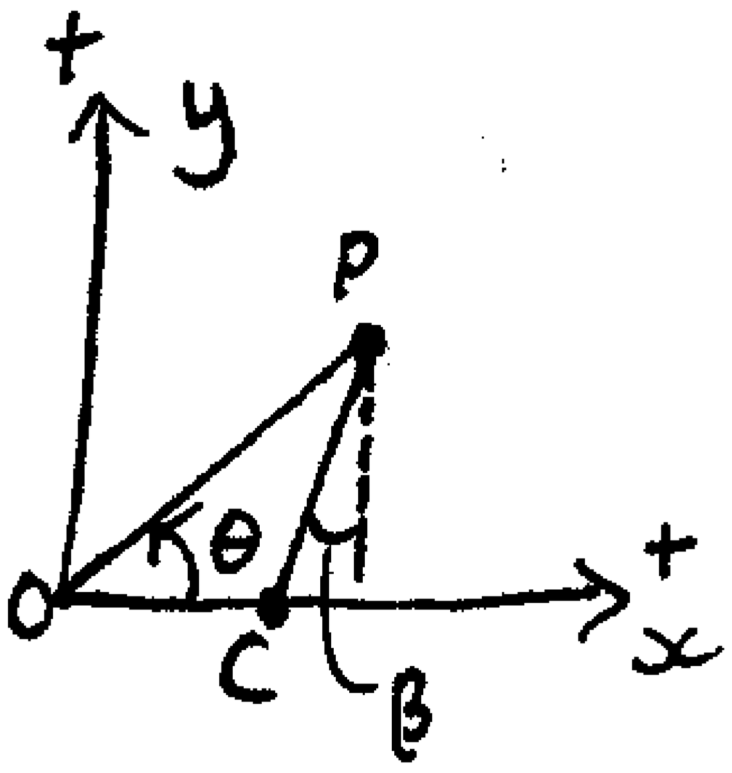
This is a 2D relative-motion problem involving acceleration (engineering mechanics).



A slotted carriage is free to slide back and forth using lubricated sleeves as support, as shown. A rotating link OP is the driver of this motion. The distance between O and P is d, and the slot makes an angle of β with the vertical. Derive an expression for the velocity and acceleration of the carriage.

Solution:

For this problem use a simple geometric approach rather than the usual vector equations.



$\measuredangle OCP = 90 + \beta$

$\measuredangle OPC = 90 - \beta - \theta$

$\dot{\theta} \equiv w$

$\ddot{\theta} \equiv \alpha$

β is constant

θ varies with time

From the law of sines,

$$\frac{OC}{\sin(\measuredangle OPC)} = \frac{OP}{\sin(\measuredangle OCP)}$$

Substitute known quantities:

$$\frac{OC}{\sin(90-\beta-\theta)} = \frac{d}{\sin(90+\beta)}$$

$$OC = d\,\frac{\sin(90-\beta-\theta)}{\sin(90+\beta)}$$

The velocity of the carriage is equal to $\frac{d(OC)}{dt}$.

(OC varies with time, so it can be differentiated with respect to time)

$$\frac{d(OC)}{dt} = \frac{d\cos(90-\beta-\theta)(-\dot{\theta})}{\sin(90+\beta)}$$

$$\Rightarrow \frac{d(OC)}{dt} = -\frac{wd\sin(\beta+\theta)}{\cos\beta} \quad \text{(answer)}$$

The acceleration of the carriage is equal to $\frac{d^2(OC)}{dt^2}$.

$$\frac{d^2(OC)}{dt^2} = -\left(\frac{\dot{w}d\sin(\beta+\theta)}{\cos\beta} + \frac{wd\cos(\beta+\theta)}{\cos\beta}\dot{\theta}\right)$$

$$\Rightarrow \frac{d^2(OC)}{dt^2} = -\frac{\alpha d\sin(\beta+\theta)}{\cos\beta} - \frac{w^2 d\cos(\beta+\theta)}{\cos\beta} \quad \text{(answer)}$$