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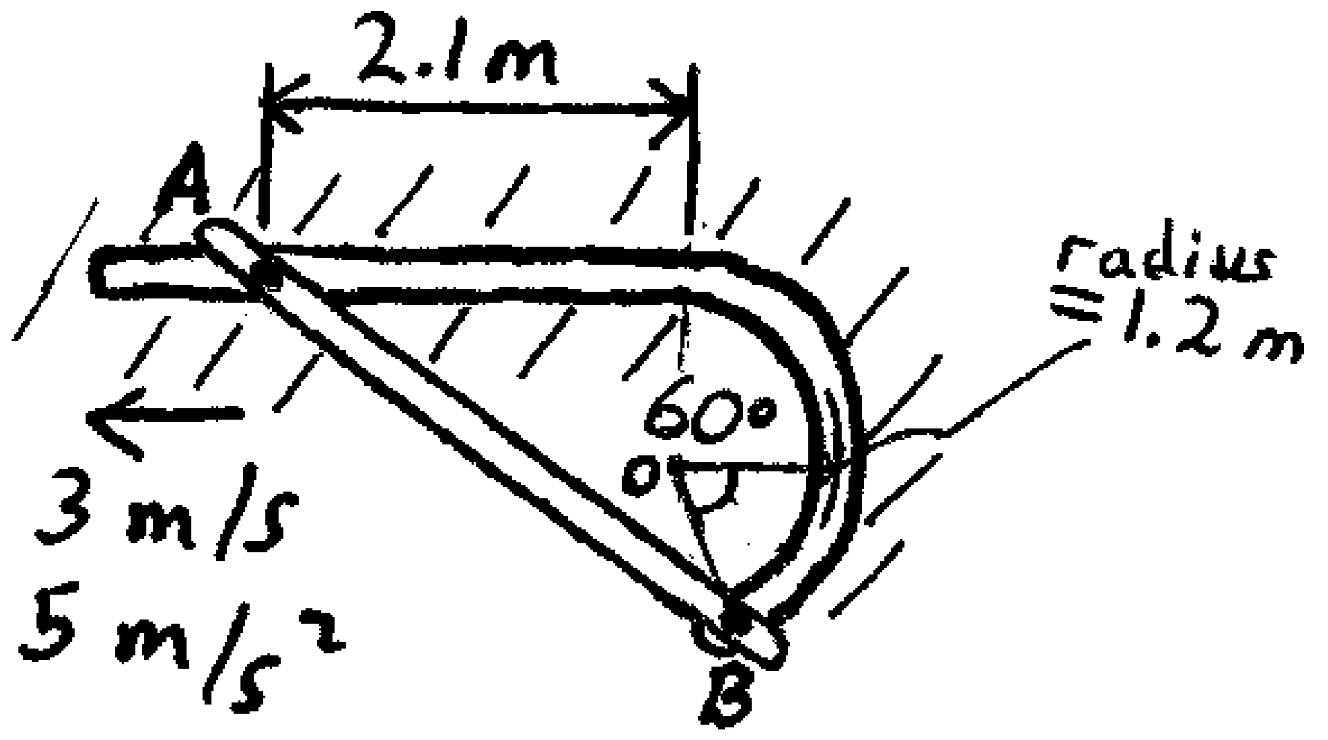
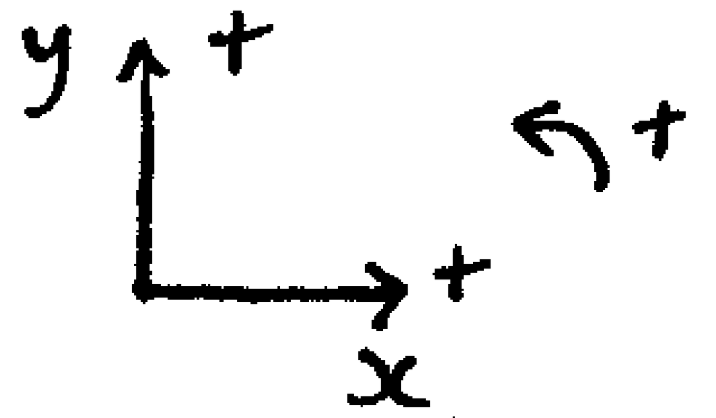
This is a 2D relative-motion problem involving acceleration (engineering mechanics).



The rod AB has endpoints at A and B which are free to move in the groove shown. Calculate the angular velocity and angular acceleration of the rod at the instant shown.

<u>Solution:</u>



First, $\vec{v}_B = \vec{v}_A + \vec{w} \times \vec{r}_{B/A}$

Substitute known quantities:

v_B is the tangential speed of point B along circular groove.

$$v_B \sin 60° \hat{i} + v_B \cos 60° \hat{j} = -3\hat{i} + w\hat{k} \times \left((2.1 + 1.2\sin 30°)\hat{i} - (1.2 + 1.2\cos 30°)\hat{j} \right) \quad (I)$$

Note that the velocity of point B is in a direction that is purely tangential to the circular groove (ie. tangent to travel path)

Next, $\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - w^2 \vec{r}_{B/A}$

Substitute known quantities:

a_B is the acceleration of point B along circular groove.

tangential component of acceleration

centripetal component of acceleration

$$a_B \sin 60^\circ \hat{i} + a_B \cos 60^\circ \hat{j} - \frac{v_B^2}{1.2} \cos 60^\circ \hat{i} + \frac{v_B^2}{1.2} \sin 60^\circ \hat{j}$$

$$= -5\hat{i} + \alpha \hat{k} \times \left((2.1 + 1.2 \sin 30^\circ)\hat{i} - (1.2 + 1.2 \cos 30^\circ)\hat{j} \right)$$

$$- w^2 \left((2.1 + 1.2 \sin 30^\circ)\hat{i} - (1.2 + 1.2 \cos 30^\circ)\hat{j} \right)$$

$$(II)$$

Simplify equation (I):

$$v_B \sin 60^\circ \hat{i} + v_B \cos 60^\circ \hat{j} = -3\hat{i} + w(2.1 + 1.2 \sin 30^\circ)\hat{j} + w(1.2 + 1.2 \cos 30^\circ)\hat{i}$$

Compare $(\hat{i})$ and $(\hat{j})$ components:

$(\hat{i})$: $v_B \sin 60^\circ = -3 + w(1.2 + 1.2 \cos 30^\circ)$

$(\hat{j})$: $v_B \cos 60^\circ = w(2.1 + 1.2 \sin 30^\circ)$

Solve the above two equations.

$v_B = -6.65$ m/s or 6.65 m/s ↙

$\Rightarrow$ $w = -1.23$ rad/s or 1.23 rad/s ↻ (answer)

Simplify equation (II):

$$a_B \sin 60° \hat{i} + a_B \cos 60° \hat{j} - \frac{v_B^2}{1.2} \cos 60° \hat{i} + \frac{v_B^2}{1.2} \sin 60° \hat{j}$$

$$= -5\hat{i} + \alpha(2.1 + 1.2 \sin 30°)\hat{j} + \alpha(1.2 + 1.2\cos 30°)\hat{i}$$

$$- w^2(2.1 + 1.2 \sin 30°)\hat{i}$$

$$+ w^2(1.2 + 1.2\cos 30°)\hat{j}$$

Compare $(\hat{i})$ and $(\hat{j})$ components and substitute known quantities:

$$(\hat{i}): \quad a_B \sin 60° - \frac{(6.65)^2}{1.2}\cos 60° = -5 + \alpha(1.2 + 1.2\cos 30°)$$

$$-(1.23)^2(2.1 + 1.2\sin 30°)$$

$$(\hat{j}): \quad a_B \cos 60° + \frac{(-6.65)^2}{1.2}\sin 60° = \alpha(2.1 + 1.2 \sin 30°)$$

$$+ (-1.23)^2(1.2 + 1.2\cos 30°)$$

Solve the above two equations.

$$a_B = 73.1 \ m/s^2$$

$$\Rightarrow \alpha = 24.1 \ \ rad/s^2 \ \circlearrowleft \quad \text{(answer)}$$