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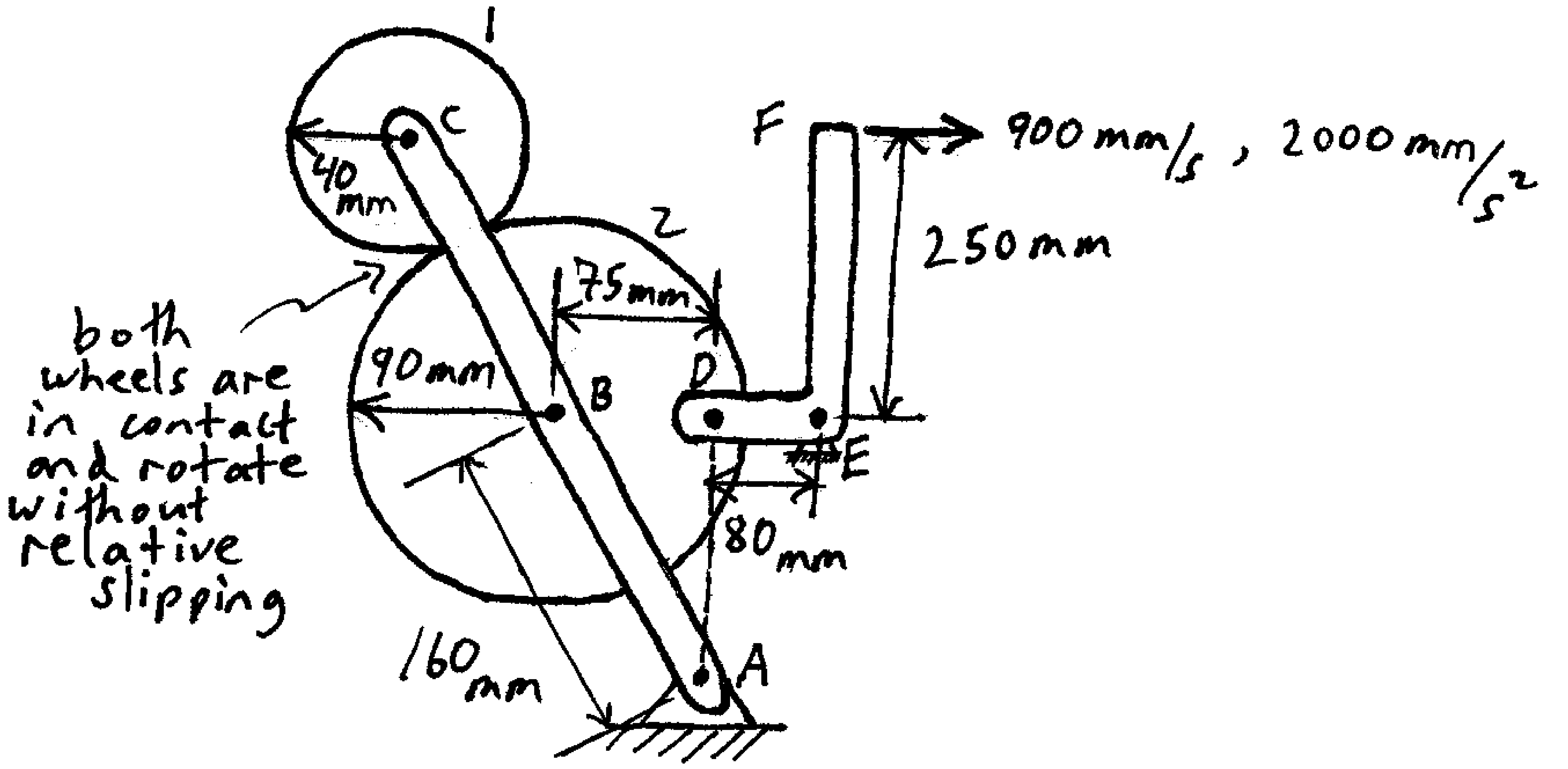
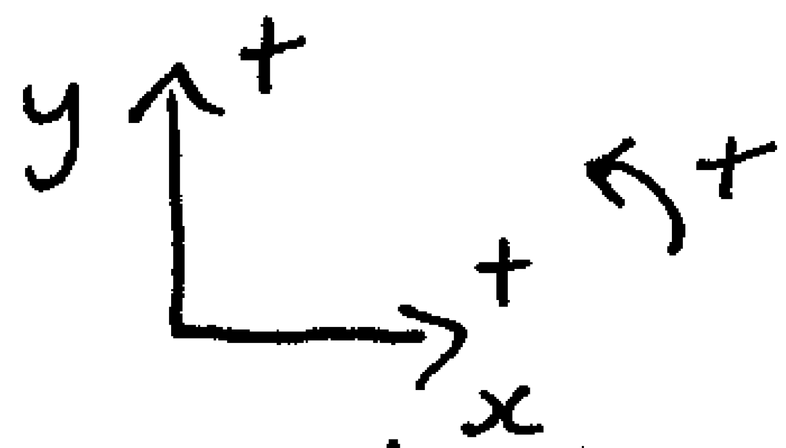
April 18, 2020

This is a 2D relative-motion analysis problem involving acceleration (engineering mechanics).



In the arrangement shown, determine the angular velocity and angular acceleration of wheel 1, wheel 2, and link AC, at the instant shown.

Solution:



coordinate frame

Note that the solution would be exactly the same if the two wheels were replaced with gears having the same radius.

The velocity of point D is:

$$\vec{V}_D = \left(\frac{80}{250}\right) 900 \text{ mm/s } \hat{j} = 288 \hat{j} \text{ mm/s} \qquad (I)$$

By geometry, the distance from A to D is: $\sqrt{160^2 - 75^2} = 141.33$ mm

The acceleration of point D is:

$$\vec{a}_D = \underbrace{\frac{\left[\left(\frac{80}{250}\right)(900)\right]^2}{80}}_{\text{centripetal component}}\hat{i} + \underbrace{\left(\frac{80}{250}\right)(2000)\hat{j}}_{\text{tangential component}}$$

$$\vec{a}_D = 1036.8\,\hat{i} + 640\,\hat{j} \quad (\text{II})$$

Next,

$$\vec{a}_D = \vec{a}_B + \vec{\alpha}_2 \times \vec{r}_{D/B} - \omega_2^2\,\vec{r}_{D/B}$$

and,

$$\vec{a}_B = \underbrace{\vec{a}_A}_{0} + \vec{\alpha}_{AC} \times \vec{r}_{B/A} - \omega_{AC}^2\,\vec{r}_{B/A}$$

Combine the above two equations:

$$\vec{a}_D = \vec{\alpha}_{AC} \times \vec{r}_{B/A} - \omega_{AC}^2\,\vec{r}_{B/A} + \vec{\alpha}_2 \times \vec{r}_{D/B} - \omega_2^2\,\vec{r}_{D/B}$$

Set equation (II) equal to the above equation and substitute known quantities:

$$1036.8\,\hat{i} + 640\,\hat{j} = \alpha_{AC}\hat{k} \times (-75\hat{i} + 141.33\hat{j}) - \omega_{AC}^2(-75\hat{i} + 141.33\hat{j}) + \alpha_2\hat{k} \times 75\hat{i} - \omega_2^2\,75\hat{i} \quad (\text{III})$$

We need additional equations involving velocity.

$$\vec{v}_D = \vec{v}_B + \vec{w}_2 \times \vec{r}_{D/B}$$

and, $\vec{v}_B = \underbrace{\vec{v}_A}_{0} + \vec{w}_{AC} \times \vec{r}_{B/A}$

Combine the above two equations:

$$\vec{v}_D = \vec{w}_{AC} \times \vec{r}_{B/A} + \vec{w}_2 \times \vec{r}_{D/B}$$

Set equation (I) equal to the above equation and substitute known quantities:

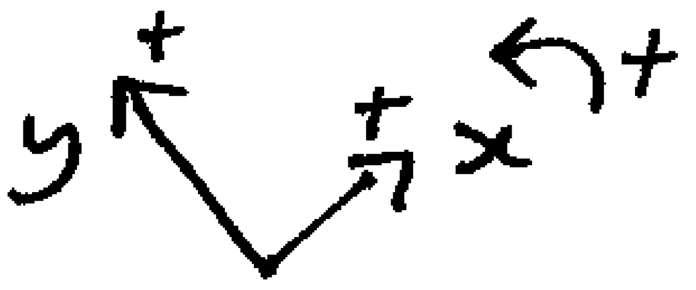
$$288\,\hat{j} = w_{AC}\,\hat{k} \times (-75\,\hat{i} + 141.33\,\hat{j})$$
$$+\, w_2\,\hat{k} \times 75\,\hat{i}$$

Simplify and solve for w_{AC} and w_2:

$$288\,\hat{j} = -75\,w_{AC}\,\hat{j} - 141.33\,w_{AC}\,\hat{i} + 75\,w_2\,\hat{j}$$

$\Longrightarrow$ From this, $w_{AC} = 0$ (answer) and $w_2 = 3.84$ rad/s ↺ (answer)

For the purpose of determining w_1, align the xy-coordinate frame so that the y-axis lies along link AC:



(for convenience)

Now, at the contact point between the two wheels (call this point P), both wheels have the same velocity (which is tangent to both wheels at this point of contact).

Then, $\vec{V}_P = \vec{V}_C + \vec{w}_1 \times \vec{r}_{P/C}$
$\vec{V}_C = \underbrace{\vec{V}_A}_{0} + \underbrace{\vec{w}_{AC}}_{0} \times \vec{r}_{C/A}$ } for wheel 1

Therefore, $\vec{V}_P = \vec{w}_1 \times \vec{r}_{P/C}$

$\vec{V}_P = w_1 \hat{k} \times (-40\hat{j})$

$\vec{V}_P = 40 w_1 \hat{i}$ (IV)

Next, $\vec{V}_P = \vec{V}_B + \vec{w}_2 \times \vec{r}_{P/B}$
$\vec{V}_B = \underbrace{\vec{V}_A}_{0} + \underbrace{\vec{w}_{AC}}_{0} \times \vec{r}_{B/A}$ } for wheel 2

Therefore, $\vec{V}_P = w_2 \hat{k} \times (90\hat{j})$

$\vec{V}_P = -90 w_2 \hat{i}$ (V)

Equate (IV) and (V):

$40 w_1 \hat{i} = -90 w_2 \hat{i}$

$w_1 = -\frac{9}{4} w_2 = -\frac{9}{4}(3.84)$

$\Longrightarrow$ $w_1 = -8.64$ rad/s (answer)

(or 8.64 rad/s ↻)

Next, simplify (III) and solve for α_{AC} and α_2 (also substituting $\omega_{AC} = 0$ and $\omega_2 = 3.84$)

$$1036.8\,\hat{i} + 640\,\hat{j} = -75\alpha_{AC}\,\hat{j} - 141.33\alpha_{AC}\,\hat{i} + 75\alpha_2\,\hat{j} - (3.84)^2(75)\,\hat{i}$$

By comparsion of ($\hat{i}$) components:

$$1036.8 = -141.33\alpha_{AC} - (3.84)^2(75)$$

$$\Rightarrow \alpha_{AC} = -15.16 \text{ rad/s}^2 \quad \text{(answer)}$$

$$\left(\text{or } 15.16 \text{ rad/s}^2 \circlearrowright\right)$$

By comparison of ($\hat{j}$) components:

$$640 = -75\alpha_{AC} + 75\alpha_2$$

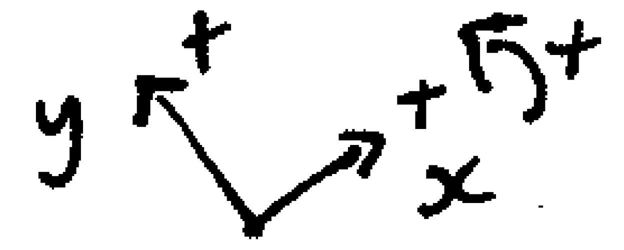
$$\Rightarrow \alpha_2 = -6.628 \text{ rad/s}^2 \quad \text{(answer)}$$

$$\left(\text{or } 6.628 \text{ rad/s}^2 \circlearrowright\right)$$

At the contact point P between the wheels, both wheels have the same tangential acceleration (tangent to both wheels at point P).

Then, $\vec{a}_P = \vec{a}_C + \vec{\alpha}_1 \times \vec{r}_{P/C}$
$\vec{a}_C = \underbrace{\vec{a}_A}_{0} + \vec{\alpha}_{AC} \times \vec{r}_{C/A}$

for wheel 1, tangential component of acceleration



y-axis aligned with link AC (for convenience)

Therefore, $\vec{a}_P = \vec{\alpha}_{AC} \times \vec{r}_{C/A} + \vec{\alpha}_1 \times \vec{r}_{P/C}$ (VI)

Next, $\vec{a}_P = \vec{a}_B + \vec{\alpha}_2 \times \vec{r}_{P/B}$
$\vec{a}_B = \underbrace{\vec{a}_A}_{0} + \vec{\alpha}_{AC} \times \vec{r}_{B/A}$
} for wheel 2, tangential component of acceleration

Therefore, $\vec{a}_P = \vec{\alpha}_{AC} \times \vec{r}_{B/A} + \vec{\alpha}_2 \times \vec{r}_{P/B}$ (VII)

Equate (VI) and (VII):

$$\vec{\alpha}_{AC} \times \vec{r}_{C/A} + \vec{\alpha}_1 \times \vec{r}_{P/C} = \vec{\alpha}_{AC} \times \vec{r}_{B/A} + \vec{\alpha}_2 \times \vec{r}_{P/B}$$

Substitute known quantities:

$$-15.16\hat{k} \times 290\hat{j} + \alpha_1 \hat{k} \times (-40\hat{j})$$
$$= -15.16\hat{k} \times 160\hat{j} - 6.628\hat{k} \times 90\hat{j}$$

$\Rightarrow$ Solving, $\alpha_1 = -34.4 \text{ rad/s}^2$ (answer)

(or $34.4 \text{ rad/s}^2 \downarrow$)