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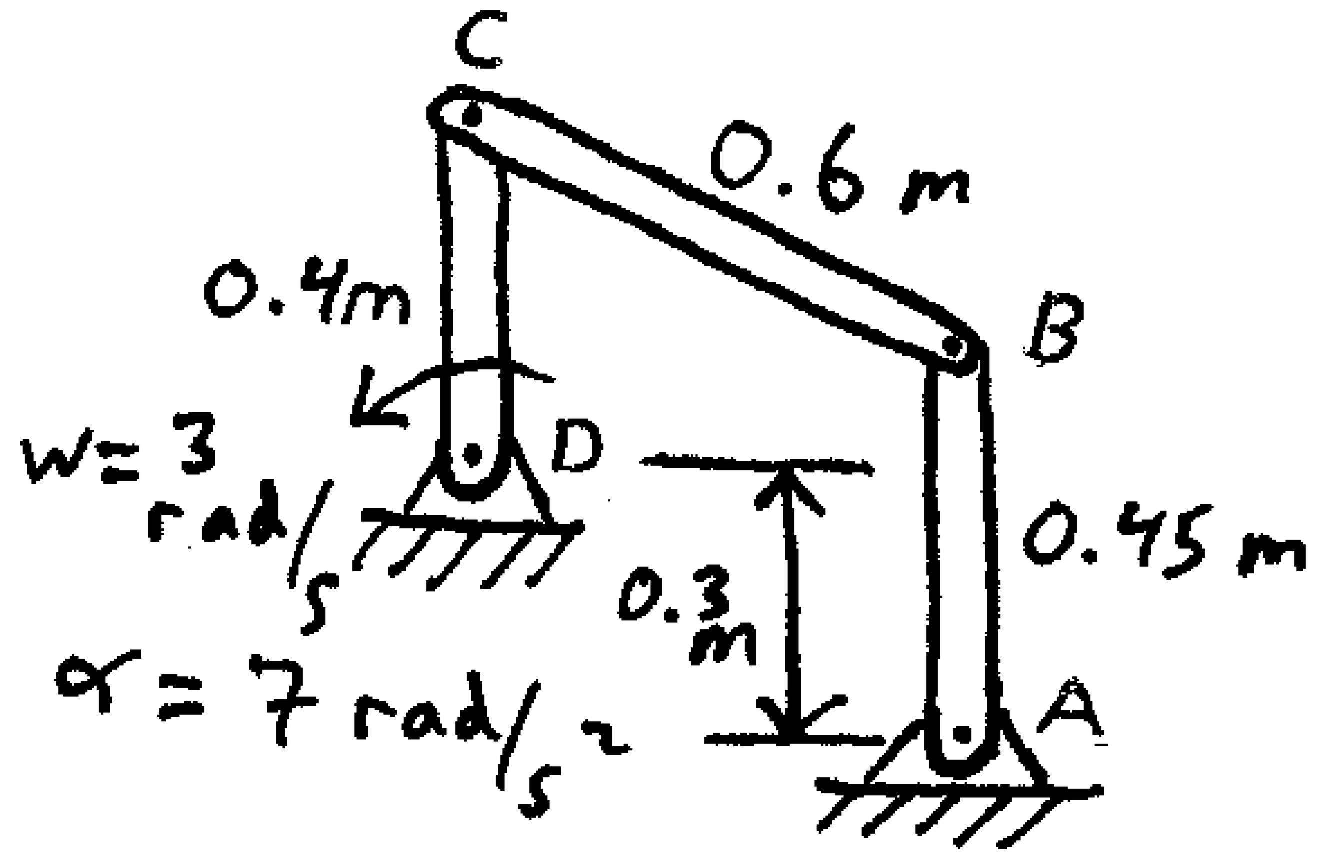
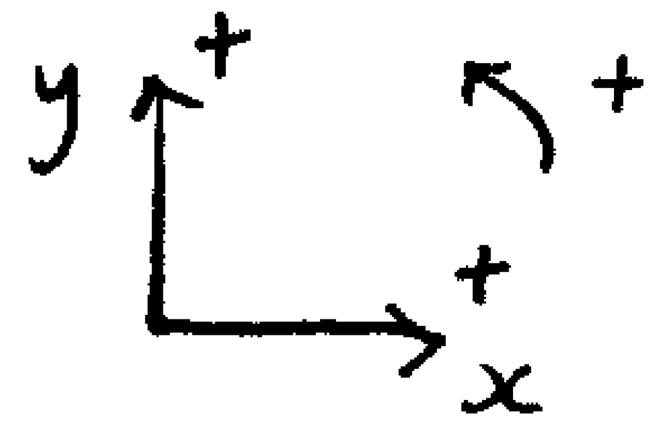
April 17, 2020

This is a 2D relative-motion analysis problem involving acceleration (engineering mechanics).

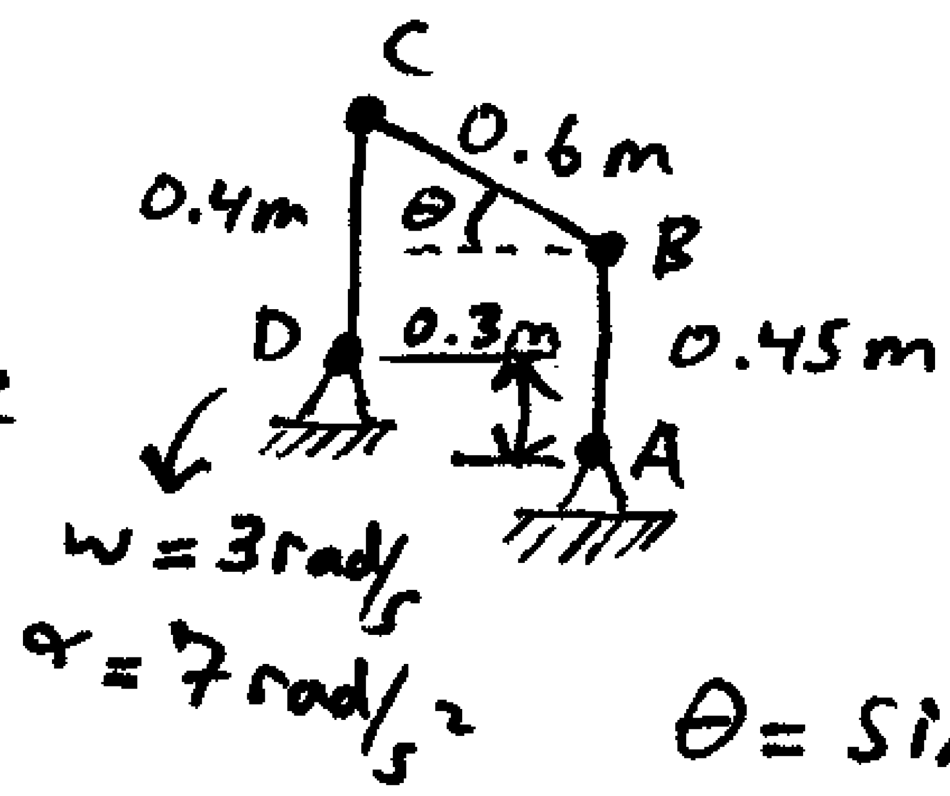


For the linkage arrangement shown, what is the angular acceleration of links AB and BC at the instant shown?

Solution:



coordinate Frame



First,

$$\vec{a}_C = \underbrace{\vec{a}_D}_{0} + \vec{\alpha} \times \vec{r}_{C/D} - w^2 \vec{r}_{C/D}$$

$\theta = \sin^{-1}\left(\frac{0.25}{0.6}\right)$

$\theta = 24.62°$

$$\Rightarrow \vec{a}_C = 7\hat{k} \times 0.4\hat{j} - (3)^2(0.4)\hat{j} \quad (I)$$

Next,

$$\vec{a}_B = \underbrace{\vec{a}_A}_{0} + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - w_{AB}^2 \vec{r}_{B/A}$$

$$\Rightarrow \vec{a}_B = \alpha_{AB}\hat{k} \times 0.45\hat{j} - \omega_{AB}^2\, 0.45\hat{j} \qquad (II)$$

Next,

$$\vec{a}_C = \vec{a}_B + \vec{\alpha}_{BC} \times \vec{r}_{C/B} - \omega_{BC}^2\, \vec{r}_{C/B}$$

sub. (II) $\Rightarrow \vec{a}_C = \alpha_{AB}\hat{k} \times 0.45\hat{j} - \omega_{AB}^2\, 0.45\hat{j}$

$$+ \alpha_{BC}\hat{k} \times (-0.6\cos\theta\,\hat{i} + 0.6\sin\theta\,\hat{j})$$

$$- \omega_{BC}^2 (-0.6\cos\theta\,\hat{i} + 0.6\sin\theta\,\hat{j})$$

$$(III)$$

We need additional equations involving velocity.

First, $\vec{v}_C = \underset{0}{\vec{v}_D} + \vec{\omega} \times \vec{r}_{C/D}$

$$\Rightarrow \vec{v}_C = 3\hat{k} \times 0.4\hat{j} \qquad (IV)$$

Next, $\vec{v}_B = \overset{0}{\vec{v}_A} + \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$$\Rightarrow \vec{v}_B = \omega_{AB}\hat{k} \times 0.45\hat{j} \qquad (V)$$

Next, $\vec{v}_C = \vec{v}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B}$

sub. (V) $\Rightarrow \vec{v}_C = \omega_{AB}\hat{k} \times 0.45\hat{j} + \omega_{BC}\hat{k} \times (-0.6\cos\theta\,\hat{i} + 0.6\sin\theta\,\hat{j})$

$$(VI)$$

Equations (IV) and (VI) are equal so,

$$3\hat{k} \times 0.4\hat{j} = w_{AB}\hat{k} \times 0.45\hat{j} + w_{BC}\hat{k} \times (-0.6\cos\theta\,\hat{i} + 0.6\sin\theta\,\hat{j})$$

$$-(3)(0.4)\hat{i} = -w_{AB}(0.45)\hat{i} - w_{BC}(0.6)\cos\theta\,\hat{j} - w_{BC}(0.6)\sin\theta\,\hat{i}$$

Compare $\hat{i}$ and $\hat{j}$ components:

$$(\hat{i}):\ -(3)(0.4) = -w_{AB}(0.45) - w_{BC}(0.6)\sin\theta$$

and

$$(\hat{j}):\ 0 = -w_{BC}(0.6)\cos\theta$$

$$w_{BC} = 0$$

substitute this into previous equation and solve for w_{AB}:

$$w_{AB} = 2.667 \text{ rad/s}$$

From before, equations (I) and (III) are equal so,

$$7\hat{k} \times 0.4\hat{j} - (3)^2(0.4)\hat{j} = \alpha_{AB}\hat{k} \times 0.45\hat{j} - (2.667)^2 0.45\hat{j} + \alpha_{BC}\hat{k} \times (-0.6\cos\theta\,\hat{i} + 0.6\sin\theta\,\hat{j})$$

$$-(7)(0.4)\hat{i} - (3)^2(0.4)\hat{j} = -\alpha_{AB}(0.45)\hat{i} - (2.667)^2 0.45\hat{j} - \alpha_{BC}(0.6)\cos\theta\,\hat{j} - \alpha_{BC}(0.6)\sin\theta\,\hat{i}$$

Compare $\hat{i}$ and $\hat{j}$ components:

$$(\hat{i}): \; -(7)(0.4) = -\alpha_{AB}(0.45) - \alpha_{BC}(0.6)\sin 24.62^\circ$$

$$(\hat{j}): \; -(3)^2(0.4) = -(2.667)^2 0.45 - \alpha_{BC}(0.6)\cos 24.62^\circ$$

From the above equation:

$\alpha_{BC} = 0.732 \text{ rad/s}^2$ (answer)

Substitute this into the previous equation and solve for α_{AB}:

$\alpha_{AB} = 5.82 \text{ rad/s}^2$ (answer)