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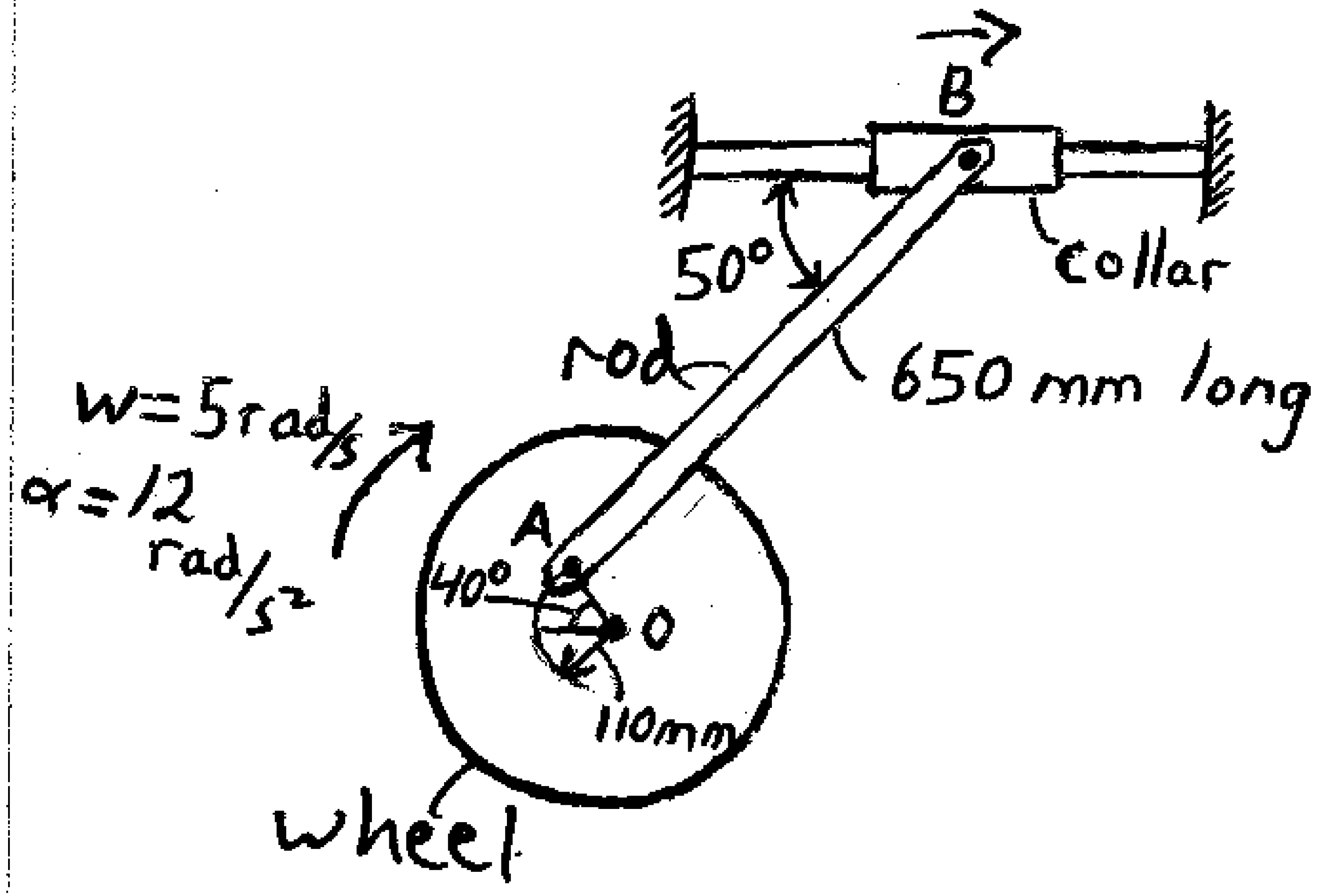
April 16, 2020

This is a 2D relative-motion analysis problem involving acceleration (engineering mechanics).



At the instant shown a wheel and rod mechanism is in motion with dimensions and angular motions as shown. Calculate the acceleration of the collar B at this instant.

<u>Solution:</u>

y^{+} ↑, $\curvearrowleft +$, $\rightarrow x^{+}$

coordinate frame

First,

$$\vec{a}_A = \underbrace{\vec{a}_o}_{0} + \vec{\alpha} \times \vec{r}_{A/O} - w^2 \vec{r}_{A/O}$$

$$\Rightarrow \vec{a}_A = -12\hat{k} \times (-110\cos 40°\,\hat{i} + 110\sin 40°\,\hat{j})$$
$$- 5^2(-110\cos 40°\,\hat{i} + 110\sin 40°\,\hat{j}) \quad (I)$$

Next,

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - w_{AB}^2\,\vec{r}_{B/A}$$

$$\Rightarrow \vec{a}_B = -12\hat{k} \times (-110\cos 40^\circ \hat{i} + 110\sin 40^\circ \hat{j})$$
$$- 5^2(-110\cos 40^\circ \hat{i} + 110\sin 40^\circ \hat{j})$$
$$+ \alpha_{AB}\hat{k} \times (650\cos 50^\circ \hat{i} + 650\sin 50^\circ \hat{j})$$
$$- w_{AB}^2 (650\cos 50^\circ \hat{i} + 650\sin 50^\circ \hat{j})$$

$$\Rightarrow \vec{a}_B = (12)(110\cos 40^\circ)\hat{j} + (12)(110\sin 40^\circ)\hat{i}$$
$$+ (25)(110\cos 40^\circ)\hat{i} - (25)(110\sin 40^\circ)\hat{j} \quad (A)$$
$$+ \alpha_{AB}(650\cos 50^\circ)\hat{j} - \alpha_{AB}(650\sin 50^\circ)\hat{i}$$
$$- w_{AB}^2(650\cos 50^\circ)\hat{i} - w_{AB}^2(650\sin 50^\circ)\hat{j}$$

Since collar B is constrained to move strictly horizontally, then the y-component of acceleration is 0. This means that:

$$(12)(110\cos 40^\circ) - (25)(110\sin 40^\circ)$$
$$+ \alpha_{AB}(650\cos 50^\circ) - w_{AB}^2(650\sin 50^\circ) = 0 \quad (B)$$

There are 2 unknowns in this equation, α_{AB} and w_{AB}. We need another equation to solve for them.

$$\vec{V}_A = \underbrace{\vec{V}_o}_{0} + \vec{w} \times \vec{r}_{A/O}$$

$$\Rightarrow \vec{V}_A = -5\hat{k} \times (-110\cos 40^\circ \hat{i} + 110\sin 40^\circ \hat{j})$$

Next,

$$\vec{V}_B = \vec{V}_A + \vec{w}_{AB} \times \vec{r}_{B/A}$$

$$\Rightarrow \vec{V}_B = -5\hat{k} \times (-110\cos 40° \hat{i} + 110\sin 40° \hat{j})$$

$$+ w_{AB}\hat{k} \times (650\cos 50° \hat{i} + 650\sin 50° \hat{j})$$

$$\Rightarrow \vec{V}_B = (5)(110\cos 40°)\hat{j} + (5)(110\sin 40°)\hat{i}$$

$$+ w_{AB}(650\cos 50°)\hat{j} - w_{AB}(650\sin 50°)\hat{i}$$

Since collar B is constrained to move strictly horizontally, then the y-component of velocity is 0. This means that:

$$(5)(110\cos 40°) + w_{AB}(650\cos 50°) = 0$$

Solve for $w_{AB} = -1.008$ rad/s

Substitute this into equation (B) and solve for α_{AB}:

$$\alpha_{AB} = 3.022 \text{ rad/s}^2$$

From equation (A):

$$\vec{a}_B = \Big[(12)(110\sin 40°) + (25)(110\cos 40°) - \alpha_{AB}(650\sin 50°) - w_{AB}^2(650\cos 50°)\Big]\hat{i}$$

substitute w_{AB} and α_{AB}:

$\vec{a}_B = \underline{\underline{1025.8}}$ mm/s^2 (answer)