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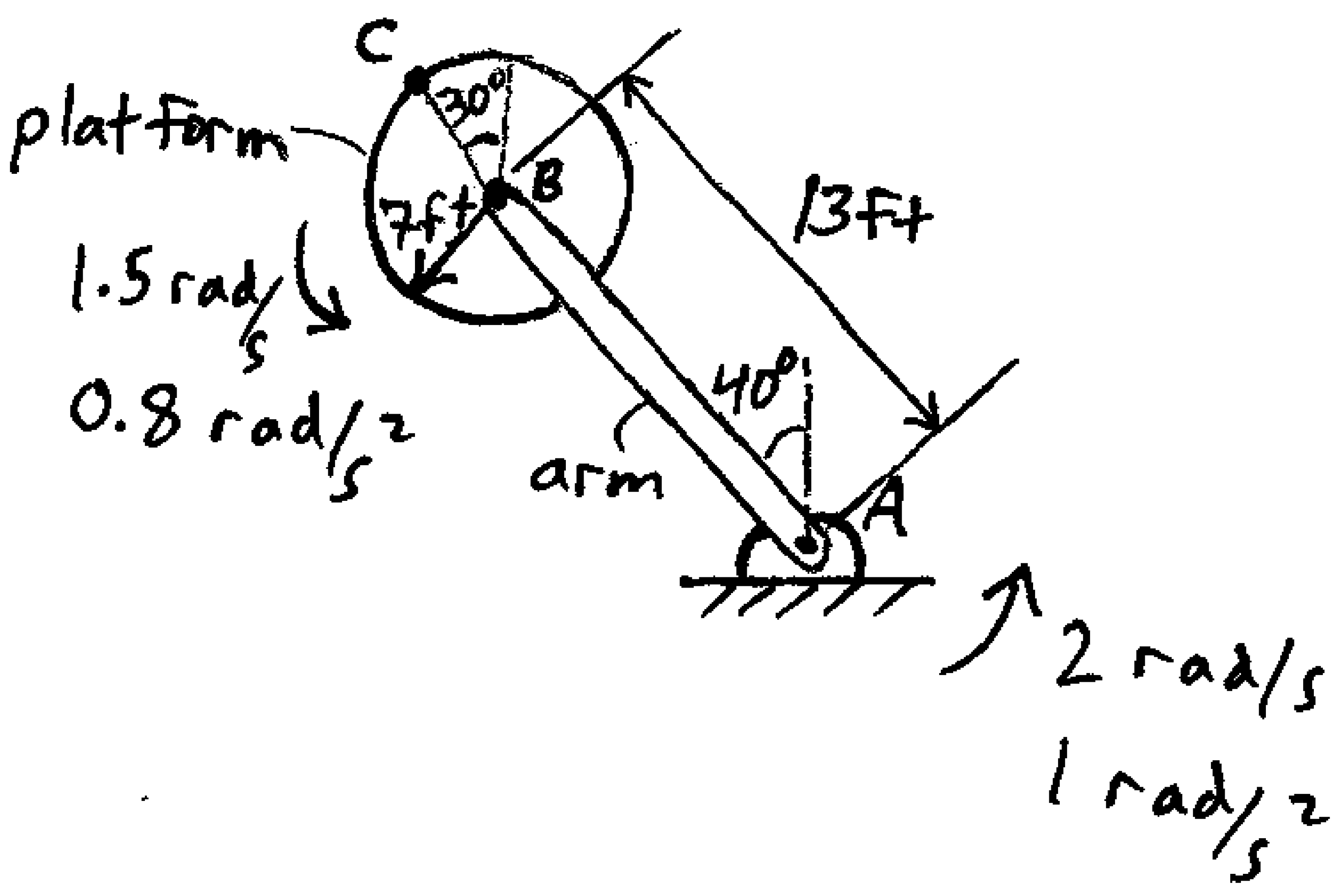
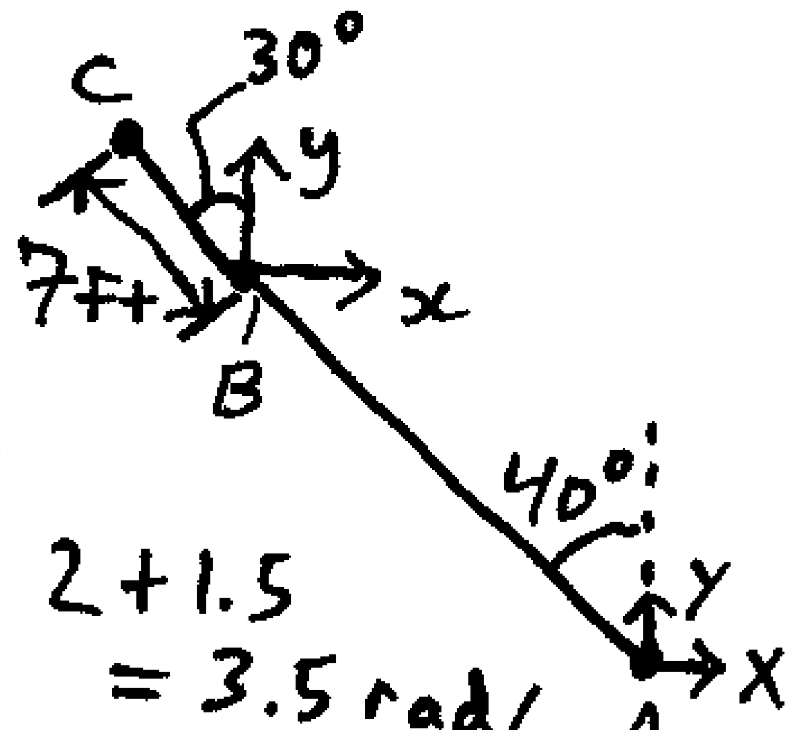
This is a 2D relative-motion analysis problem involving rotating axes (engineering mechanics).



In an amusement park ride, the main arm is rotating about A with an angular velocity of $w_a = 2$ rad/s and an angular acceleration of $\alpha_a = 1$ rad/s². A platform is attached to the main arm and rotates about point B due to a motor which rotates with a rotational speed of 1.5 rad/s and a rotational acceleration of 0.8 rad/s². A person is sitting on the platform at point C. At the instant shown, what is the velocity and acceleration of the person?

Solution:



The motor angular velocity/acceleration must be added to the arm angular velocity/acceleration, which gives the angular velocity/acceleration of the person with respect to ground - which is what must be used!

$$w_c = 2 + 1.5 = 3.5 \text{ rad/s}$$

$$\alpha_c = 1 + 0.8 = 1.8 \text{ rad/s}^2$$

$$w_a = 2 \text{ rad/s}$$
$$\alpha_A = 1 \text{ rad/s}^2$$

angular velocity and angular acceleration of arm with respect to ground

$$\vec{V}_C = \vec{V}_B + \vec{w}_c \times \vec{r}_{C/B} + \underbrace{(\vec{V}_{C/B})_{rel}}_{0} \quad \text{I}$$

$$\vec{a}_C = \vec{a}_B + \vec{\alpha}_c \times \vec{r}_{C/B} + \vec{w}_c \times (\vec{w}_c \times \vec{r}_{C/B}) + 2\vec{w}_c \times \underbrace{(\vec{V}_{C/B})_{rel}}_{0}$$

$$+ \underbrace{(\vec{a}_{C/B})_{rel}}_{0} \quad \text{II}$$

$$\text{I} \Rightarrow \vec{V}_C = \vec{V}_B + \vec{w}_c \times \vec{r}_{C/B}$$

$$\text{II} \Rightarrow \vec{a}_c = \vec{a}_B + \vec{\alpha}_c \times \vec{r}_{C/B} + \vec{w}_c \times (\vec{w}_c \times \vec{r}_{C/B})$$

Now, $\vec{r}_{C/B} = -7\sin 30^\circ \hat{i} + 7\cos 30^\circ \hat{j}$

$$\vec{w}_c = w_c \hat{k} = 3.5\hat{k}$$

$$\vec{\alpha}_c = \alpha_c \hat{k} = 1.8\hat{k}$$

$$\vec{V}_B = \underbrace{\vec{V}_A}_{0} + \vec{w}_a \times \vec{r}_{B/A} + \underbrace{(\vec{V}_{B/A})_{rel}}_{0}$$

$$\vec{r}_{B/A} = -13\sin 40^\circ \hat{i} + 13\cos 40^\circ \hat{j}$$

$$\vec{w}_a = w_a \hat{k} = 2\hat{k}$$

$$\vec{\alpha}_a = \alpha_a \hat{k} = 1\hat{k}$$

Substitute:

$$\vec{V}_B = 2\hat{k} \times (-13\sin 40^\circ \hat{i} + 13\cos 40^\circ \hat{j})$$

Substitute into I:

$$\vec{V}_c = 2\hat{k} \times (-13\sin 40^\circ \hat{i} + 13\cos 40^\circ \hat{j}) + 3.5\hat{k} \times (-7\sin 30^\circ \hat{i} + 7\cos 30^\circ \hat{j})$$

$$\vec{V}_c = \underline{-41.135\hat{i} - 28.962\hat{j}} \text{ ft/s} \quad (\text{answer})$$

Next,

$$\vec{a}_B = \underset{0}{\underset{\downarrow}{\vec{a}_A}} + \vec{\alpha}_A + \vec{r}_{B/A} + \vec{\omega}_a \times (\vec{\omega}_a \times \vec{r}_{B/A}) + 2\vec{\omega}_a \times \overset{0}{\overset{\nearrow}{(\vec{v}_{B/A})_{rel}}} + \underset{0}{\underset{\downarrow}{(\vec{a}_{B/A})_{rel}}}$$

Substitute:

$$\vec{a}_B = 1\hat{k} \times (-13\sin 40^\circ \hat{i} + 13\cos 40^\circ \hat{j}) + 2\hat{k} \times (2\hat{k} \times (-13\sin 40^\circ \hat{i} + 13\cos 40^\circ \hat{j}))$$

$$\vec{a}_B = 23.466\hat{i} - 48.191\hat{j}$$

Substitute into (II):

$$\vec{a}_c = 23.466\hat{i} - 48.191\hat{j} + 1.8\hat{k} \times (-7\sin 30^\circ \hat{i} + 7\cos 30^\circ \hat{j}) + 3.5\hat{k} \times (3.5\hat{k} \times (-7\sin 30^\circ \hat{i} + 7\cos 30^\circ \hat{j}))$$

$$\vec{a}_c = \underline{\underline{55.429\hat{i} - 128.753\hat{j}}} \ \text{ft}/\text{s}^2 \quad (\text{answer})$$