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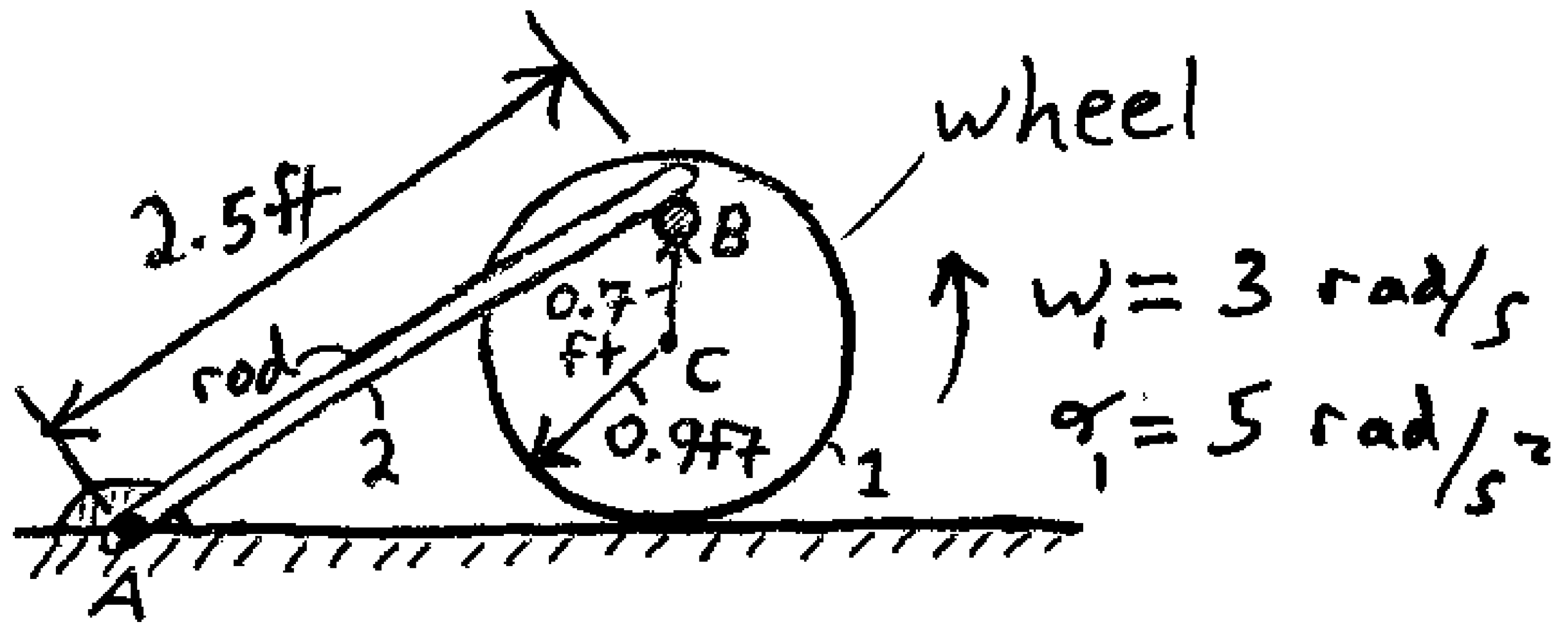
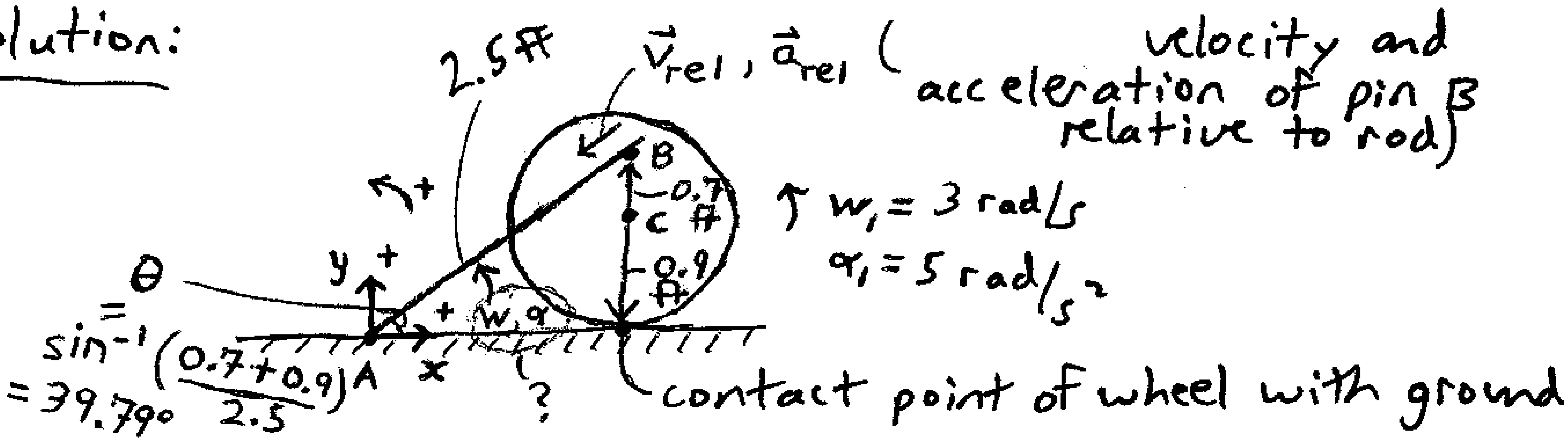
This is a 2D relative-motion analysis problem involving rotating axes (engineering mechanics).



A wheel is rolling as shown without slipping. The pin B on the wheel is pushing against and sliding along the rod which is free to pivot about point A. What is the angular velocity and angular acceleration of the rod?

Solution:



Since the wheel is rolling without slipping then:

$$\vec{V}_B = -(0.9+0.7)\,\omega_1\,\hat{i}$$

$$\vec{V}_B = -1.6\,\omega_1\,\hat{i} = -4.8\,\hat{i} \quad (1)$$

(since B, C, and the contact point with ground lie on a straight line)

and

$$\vec{a}_B = -(0.9+0.7)\,\alpha_1\,\hat{i} \quad \text{\} tangential component}$$

$$-\,0.7\,\omega_1^2\,\hat{j} \quad \text{\} centripetal component}$$

$$\vec{a}_B = -1.6\alpha_1\,\hat{i} - 0.7\omega_1^2\,\hat{j} = -8\,\hat{i} - 6.3\,\hat{j} \quad (2)$$

Now, analyze the rod:

$$\vec{V}_B = \vec{V}_A + \vec{w} \times \vec{r}_{B/A} + (\vec{V}_{B/A})_{rel} \quad (3)$$

$\vec{V}_A = 0$ (Fixed pivot point)

$\vec{w} = w\hat{k}$ (w is the magnitude of the rod angular velocity - it is unknown)

From geometry: $\vec{r}_{B/A} = \sqrt{2.5^2 - (0.7+0.9)^2}\,\hat{i} + (0.7+0.9)\hat{j}$

$$\vec{r}_{B/A} = 1.921\,\hat{i} + 1.6\,\hat{j}$$

$$(\vec{V}_{B/A})_{rel} = \vec{V}_{rel} = -V_{rel}\cos 39.79°\,\hat{i} - V_{rel}\sin 39.79°\,\hat{j}$$

V_{rel} is the magnitude of the velocity of the pin B relative to the rod - it is unknown

Substitute (1) and other known quantities specified above into (3):

$$-4.8\,\hat{i} = w\hat{k} \times (1.921\,\hat{i} + 1.6\,\hat{j}) - V_{rel}\cos 39.79°\,\hat{i} - V_{rel}\sin 39.79°\,\hat{j}$$

$$-4.8\,\hat{i} = 1.921w\,\hat{j} - 1.6w\,\hat{i} - V_{rel}\cos 39.79°\,\hat{i} - V_{rel}\sin 39.79°\,\hat{j}$$

Compare $\hat{i}$ components:

$$-4.8 = -1.6w - V_{rel}\cos 39.79°$$

Compare $\hat{j}$ components:

$$0 = 1.921w - V_{rel}\sin 39.79°$$

Solve the above 2 equations:

$\boxed{w = 1.229 \text{ rad/s}}$ ↺ (answer)

$V_{rel} = 3.688 \text{ ft/s}$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A}) + 2\vec{\omega} \times (\vec{V}_{B/A})_{rel} + (\vec{a}_{B/A})_{rel} \quad (4)$$

$\vec{a}_A = 0$ (fixed pivot point)

$\vec{\alpha} = \alpha \hat{k}$ (α is the magnitude of the rod angular acceleration - it is unknown)

$$(\vec{a}_{B/A})_{rel} = \vec{a}_{rel} = -a_{rel} \cos 39.79° \hat{i} - a_{rel} \sin 39.79° \hat{j}$$

a_{rel} is the magnitude of the acceleration of the pin B relative to the rod - it is unknown

Substitute (2) and other known quantities specified above into (4):

$$-8\hat{i} - 6.3\hat{j} = \alpha \hat{k} \times (1.921\hat{i} + 1.6\hat{j}) + w\hat{k} \times (w\hat{k} \times (1.921\hat{i} + 1.6\hat{j})) + 2w\hat{k} \times (-V_{rel} \cos 39.79° \hat{i} - V_{rel} \sin 39.79° \hat{j}) - a_{rel} \cos 39.79° \hat{i} - a_{rel} \sin 39.79° \hat{j}$$

$$-8\hat{i}-6.3\hat{j} = 1.921\alpha\hat{j} - 1.6\alpha\hat{i}$$
$$+ \omega\hat{k}\times(1.921\omega\hat{j} - 1.6\omega\hat{i})$$
$$-2\omega v_{rel}\cos 39.79°\hat{j}$$
$$+2\omega v_{rel}\sin 39.79°\hat{i}$$
$$-a_{rel}\cos 39.79°\hat{i}$$
$$-a_{rel}\sin 39.79°\hat{j}$$

$$-8\hat{i}-6.3\hat{j} = 1.921\alpha\hat{j} - 1.6\alpha\hat{i} - 1.921\omega^2\hat{i} - 1.6\omega^2\hat{j}$$
$$-2\omega v_{rel}\cos 39.79°\hat{j} + 2\omega v_{rel}\sin 39.79°\hat{i}$$
$$-a_{rel}\cos 39.79°\hat{i} - a_{rel}\sin 39.79°\hat{j}$$

Compare $\hat{i}$ components:

$$-8 = -1.6\alpha - 1.921\omega^2 + 2\omega v_{rel}\sin 39.79° - a_{rel}\cos 39.79°$$

Compare $\hat{j}$ components:

$$-6.3 = 1.921\alpha - 1.6\omega^2 - 2\omega v_{rel}\cos 39.79° - a_{rel}\sin 39.79°$$

From before, $\omega = 1.229$ rad/s

$v_{rel} = 3.688$ ft/s

Substitute and solve the above two equations for α and a_{rel}.

$\alpha = 3.738$ rad/s² ↶ (answer)

$a_{rel} = 6.40$ ft/s²