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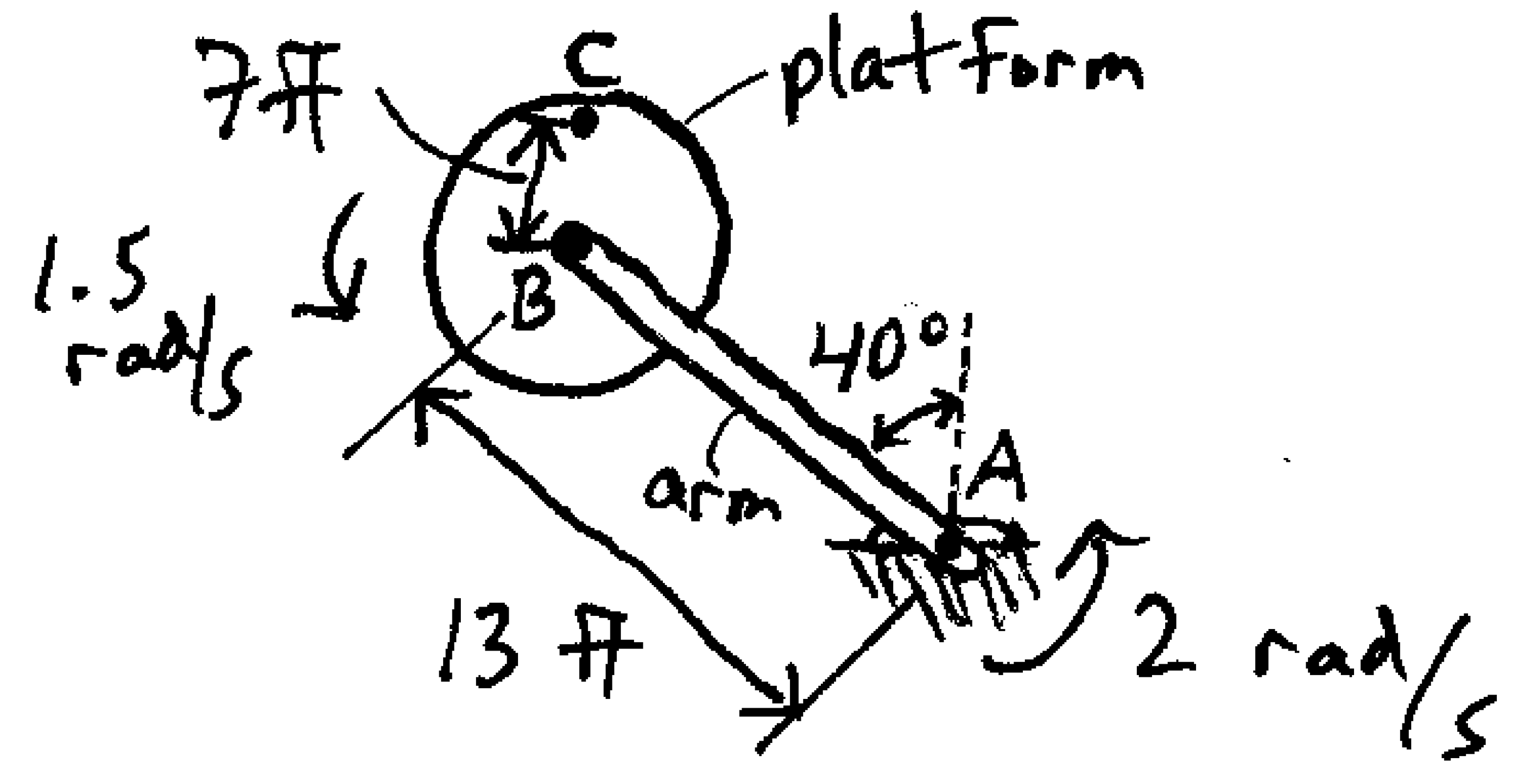
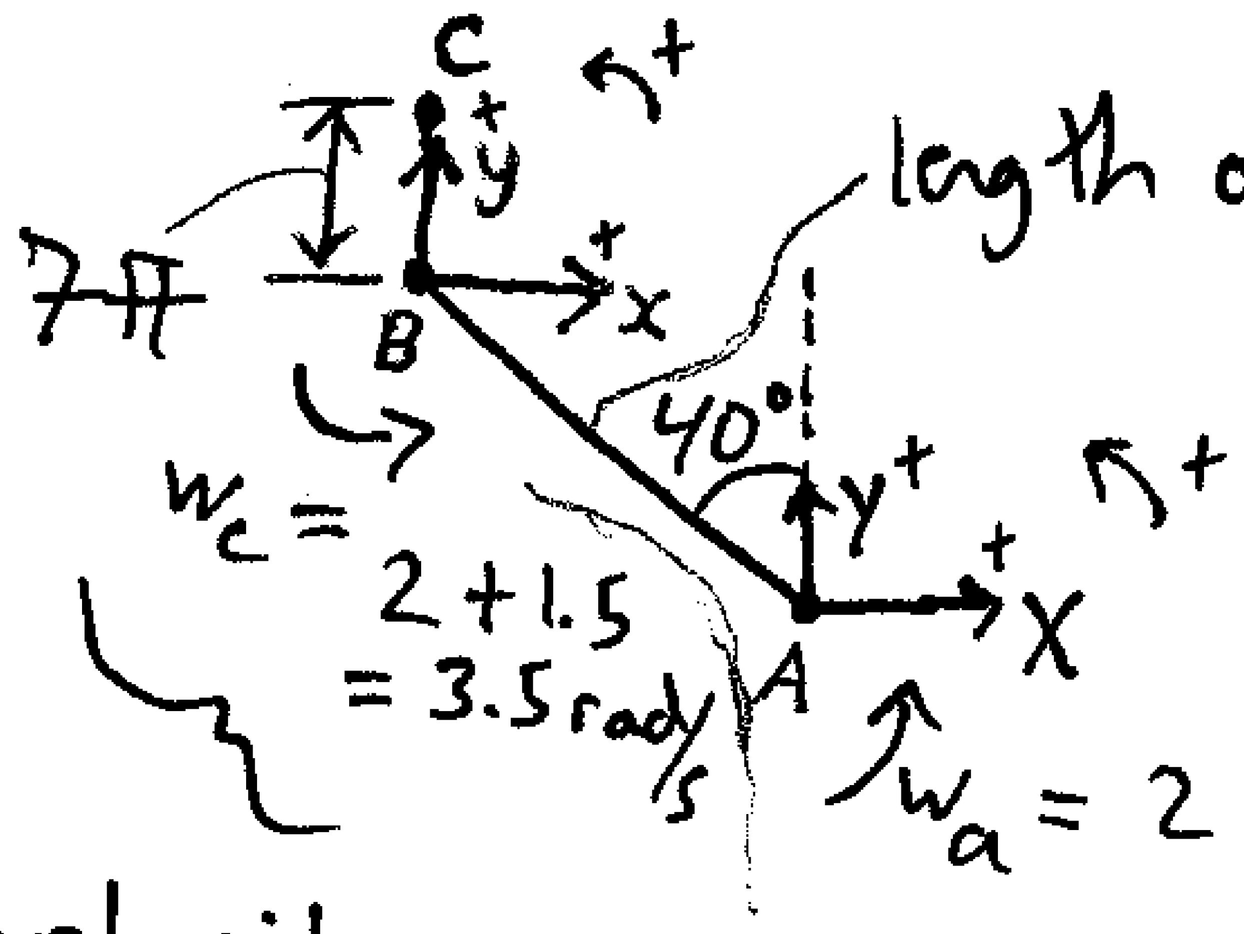
April 10, 2020

This is a 2D relative-motion analysis problem involving rotating axes (engineering mechanics).



In an amusement park ride, the main arm is rotating about A at a constant angular velocity of $w_a = 2$ rad/s. A platform is attached to the main arm and rotates about point B due to a motor which rotates at a constant rotational speed of 1.5 rad/s. A person is sitting on the platform at point C. At the instant shown, what is the velocity and acceleration of the person?

<u>Solution:</u>



length of AB = 13 ft

$w_a = 2$ rad/s (angular velocity of arm with respect to ground)

The motor angular velocity must be added to the arm angular velocity, which gives the angular velocity of the person with respect to ground - which is what must be used!

The motor angular velocity is given relative to the arm.

$$\vec{V}_c = \vec{V}_B + \vec{w}_c \times \vec{r}_{C/B} + \overset{\nearrow 0}{(\vec{V}_{C/B})_{rel}} \quad \text{(I)}$$

$$\vec{a}_c = \vec{a}_B + \underset{\downarrow 0}{\vec{\alpha}_c} \times \vec{r}_{C/B} + \vec{w}_c \times (\vec{w}_c \times \vec{r}_{C/B}) + 2\vec{w}_c \times \overset{\nearrow 0}{(\vec{V}_{C/B})_{rel}}$$

$$+ \underset{\searrow 0}{(\vec{a}_{C/B})_{rel}} \quad \text{(II)}$$

$$\text{(I)} \Rightarrow \vec{V}_c = \vec{V}_B + \vec{w}_c \times \vec{r}_{C/B}$$

$$\text{(II)} \Rightarrow \vec{a}_c = \vec{a}_B + \vec{w}_c \times (\vec{w}_c \times \vec{r}_{C/B})$$

Now, $\vec{r}_{C/B} = 7\hat{j}$

$\vec{w}_c = w_c \hat{k} = 3.5\hat{k}$ (constant, so $\vec{\alpha}_c = 0$)

$$\vec{V}_B = \underset{\downarrow 0}{\vec{V}_A} + \vec{w}_a \times \vec{r}_{B/A} + \overset{\nearrow 0}{(\vec{V}_{B/A})_{rel}}$$

$\vec{r}_{B/A} = -13\sin 40^\circ \hat{i} + 13\cos 40^\circ \hat{j}$

$\vec{w}_a = w_a \hat{k} = 2\hat{k}$ (constant, so $\vec{\alpha}_a = 0$)

Substitute:

$$\vec{V}_B = 2\hat{k} \times (-13\sin 40^\circ \hat{i} + 13\cos 40^\circ \hat{j})$$

Substitute into (I):

$$\vec{V}_c = 2\hat{k} \times (-13\sin 40^\circ \hat{i} + 13\cos 40^\circ \hat{j}) + 3.5\hat{k} \times 7\hat{j}$$

$$\vec{V}_c = -26\sin 40^\circ \hat{j} - 26\cos 40^\circ \hat{i} - 24.5\hat{i}$$

$$\vec{V}_c = \underline{\underline{-44.417\hat{i} - 16.712\hat{j}}} \ \text{ft/s} \quad \text{(answer)}$$

Next,

$$\vec{a}_B = \underbrace{\vec{a}_A}_{0} + \overbrace{\vec{\alpha}_a}^{0} \times \vec{r}_{B/A} + \vec{\omega}_a \times (\vec{\omega}_a \times \vec{r}_{B/A}) + 2\vec{\omega}_a \times \overbrace{(\vec{v}_{B/A})_{rel}}^{0} + \underbrace{(\vec{a}_{B/A})_{rel}}_{0}$$

Substitute:

$$\vec{a}_B = 2\hat{k} \times (2\hat{k} \times (-13\sin 40°\,\hat{i} + 13\cos 40°\,\hat{j}))$$

$$\vec{a}_B = 2\hat{k} \times (-26\sin 40°\,\hat{j} - 26\cos 40°\,\hat{i})$$

$$\vec{a}_B = 52\sin 40°\,\hat{i} - 52\cos 40°\,\hat{j}$$

Substitute into (II):

$$\vec{a}_c = 52\sin 40°\,\hat{i} - 52\cos 40°\,\hat{j} + 3.5\hat{k} \times (3.5\hat{k} \times 7\hat{j})$$

$$\vec{a}_c = 52\sin 40°\,\hat{i} - 52\cos 40°\,\hat{j} - 85.75\,\hat{j}$$

$$\vec{a}_c = \underline{\underline{33.425\,\hat{i} - 125.584\,\hat{j}}}\ \text{ft/s}^2 \quad (\text{answer})$$