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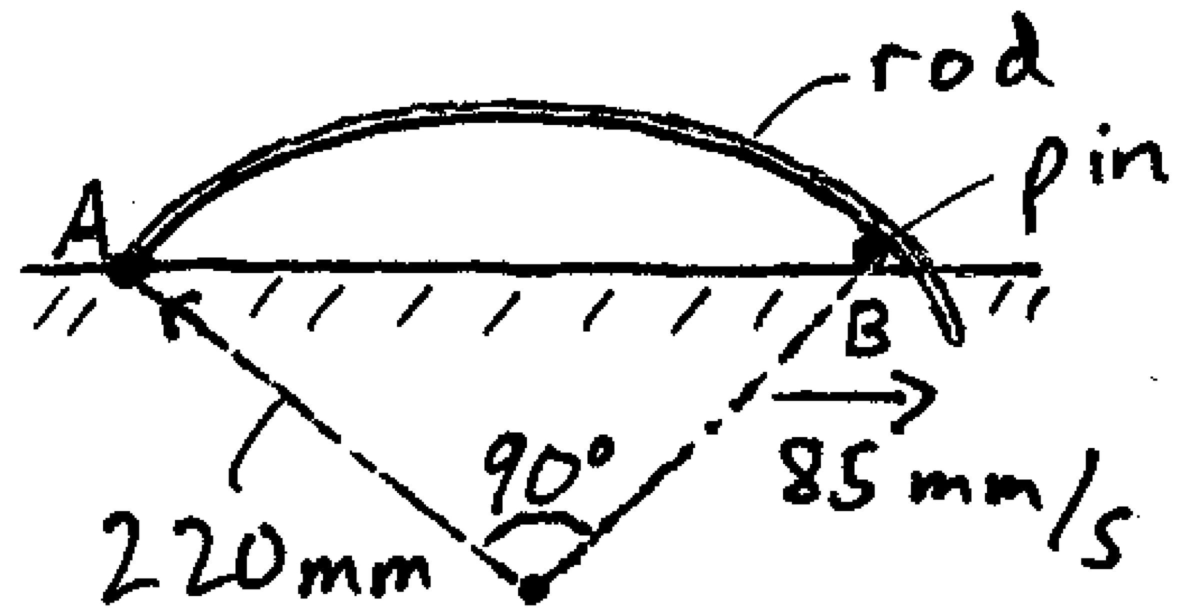
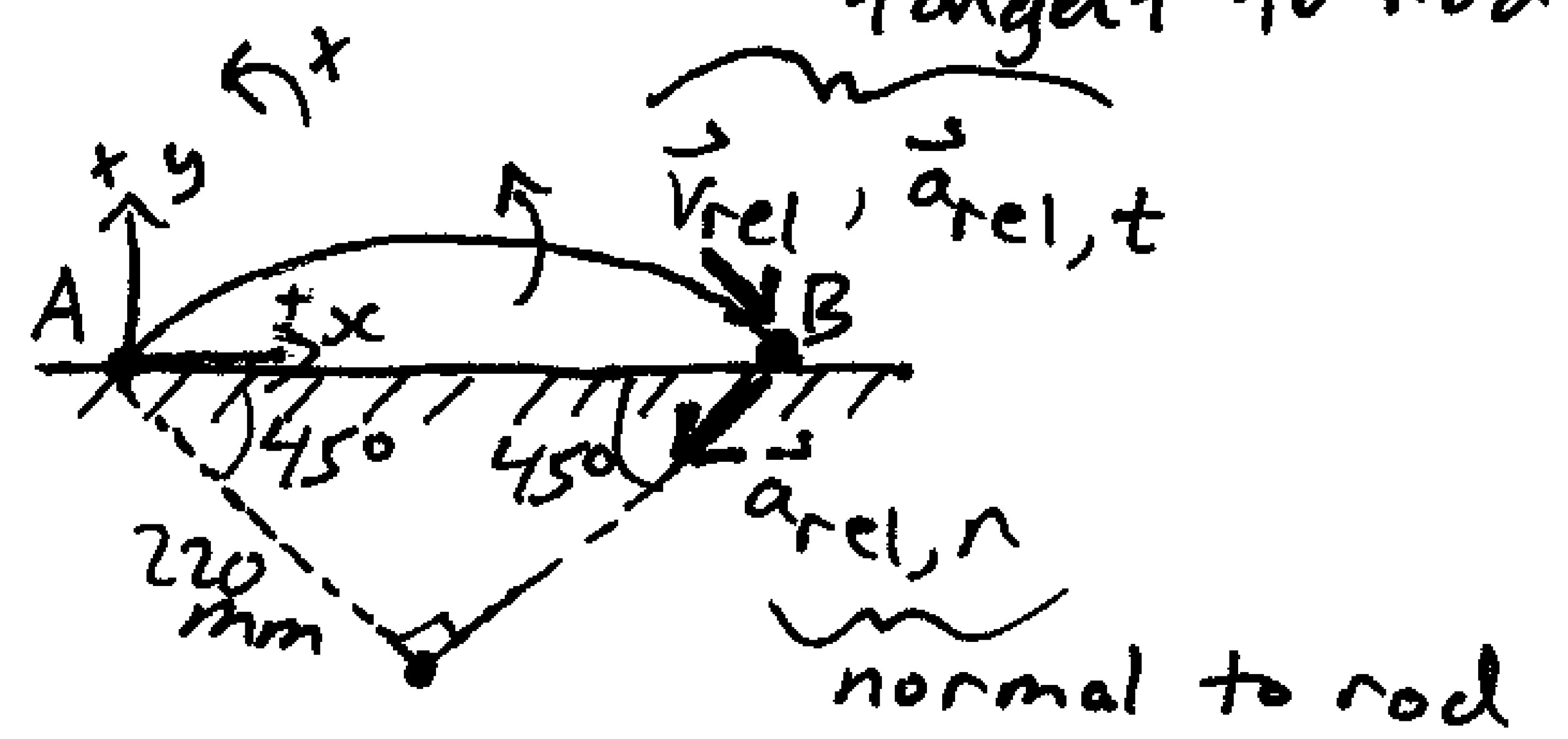
This is a 2D relative-motion analysis problem involving rotating axes (engineering mechanics).



A rod in the shape of a circular arc rotates about A such that a pin pushing against the rod, as shown, is moving to the right at a constant speed of 85 mm/s. Determine the angular velocity and angular acceleration of the rod at the instant shown. Note that the pin is sliding along the rod.

Solution:



$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A} + (\vec{v}_{B/A})_{rel} \quad \text{(I)}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A}) + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} + (\vec{a}_{B/A})_{rel} \quad \text{(II)}$$

$\vec{v}_A = 0$ (fixed pivot point)

$\vec{\omega} = \omega \hat{k}$ rad/s (ω is unknown - it is the magnitude of the angular velocity of rod)

$\vec{v}_B = 85\,\hat{i}$ mm/s (as given in problem statement)

$(\vec{v}_{B/A})_{rel} = \vec{v}_{rel} = v_{rel}\cos 45°\,\hat{i} - v_{rel}\sin 45°\,\hat{j}$

(v_{rel} is unknown - it is the magnitude of the velocity of the pin relative to the rod)

$\vec{r}_{B/A} = 2(220)\cos 45°\,\hat{i}$ mm

$\vec{a}_B = 0$ (since $\vec{v}_B$ is constant)

$\vec{a}_A = 0$ (fixed pivot point)

$\vec{\alpha} = \alpha\,\hat{k}$ rad/s² (α is unknown - it is the magnitude of the angular acceleration of rod)

$(\vec{a}_{B/A})_{rel} = \vec{a}_{rel,t} + \vec{a}_{rel,n}$

$\vec{a}_{rel,t} = a_{rel,t}\cos 45°\,\hat{i} - a_{rel,t}\sin 45°\,\hat{j}$

($a_{rel,t}$ is unknown - it is the magnitude of the tangential acceleration of the pin, relative to the rod)

$\vec{a}_{rel,n} = -a_{rel,n}\cos 45°\,\hat{i} - a_{rel,n}\sin 45°\,\hat{j}$

($a_{rel,n}$ is the magnitude of the normal acceleration of the pin, relative to the rod)

$$a_{rel,n} = \frac{(v_{rel})^2}{220}$$

(centripetal acceleration since the pin follows a circular arc along the rod)

Substitute known quantities into equations (I) and (II):

(I) $\Rightarrow 85\,\hat{i} = 0 + w\hat{k} \times 2(220)\cos 45°\,\hat{i}$

$+ V_{rel}\cos 45°\,\hat{i} - V_{rel}\sin 45°\,\hat{j}$

$\Rightarrow 85\,\hat{i} = 2(220)\cos 45°\, w\,\hat{j} - V_{rel}\sin 45°\,\hat{j}$

$+ V_{rel}\cos 45°\,\hat{i}$

By comparison:
($\hat{i}$ terms) $V_{rel}\cos 45° = 85$

$V_{rel} = 120.208$ mm/s

and

($\hat{j}$ terms) $0 = 2(220)\cos 45°\, w - V_{rel}\sin 45°$

Note that if this were negative it would mean $\vec{w}$ is in opposite direction

$\boxed{W = 0.273}$ rad/s ↺ (answer)

(II) $\Rightarrow 0 = 0 + \alpha\hat{k} \times 2(220)\cos 45°\,\hat{i}$

$+ w\hat{k} \times (w\hat{k} \times 2(220)\cos 45°\,\hat{i})$

$+ 2w\hat{k} \times (V_{rel}\cos 45°\,\hat{i} - V_{rel}\sin 45°\,\hat{j})$

$+ a_{rel,t}\cos 45°\,\hat{i} - a_{rel,t}\sin 45°\,\hat{j}$

$- \dfrac{(V_{rel})^2}{220}\cos 45°\,\hat{i} - \dfrac{(V_{rel})^2}{220}\sin 45°\,\hat{j}$

$$\Rightarrow \quad 0 = 2(220)\cos 45^\circ \alpha \hat{j} - 2(220)\cos 45^\circ \omega^2 \hat{i}$$
$$+ 2\omega v_{rel}\cos 45^\circ \hat{j} + 2\omega v_{rel}\sin 45^\circ \hat{i}$$
$$+ a_{rel,t}\cos 45^\circ \hat{i} - a_{rel,t}\sin 45^\circ \hat{j}$$
$$- \frac{(v_{rel})^2}{220}\cos 45^\circ \hat{i} - \frac{(v_{rel})^2}{220}\sin 45^\circ \hat{j}$$

By comparison:

$$(\hat{i}\text{ terms}) \quad 0 = -2(220)\cos 45^\circ \omega^2 + 2\omega v_{rel}\sin 45^\circ + a_{rel,t}\cos 45^\circ - \frac{(v_{rel})^2}{220}\cos 45^\circ$$

$$a_{rel,t} = 2(220)\omega^2 - 2\omega v_{rel} + \frac{(v_{rel})^2}{220}$$

sub. ω and v_{rel}:

$$a_{rel,t} = 2(220)(0.273)^2 - 2(0.273)(120.208) + \frac{(120.208)^2}{220}$$

$$a_{rel,t} = 32.841 \ \text{mm/s}^2$$

By comparison:

$$(\hat{j}\text{ terms}) \quad 0 = 2(220)\cos 45^\circ \alpha + 2\omega v_{rel}\cos 45^\circ - a_{rel,t}\sin 45^\circ - \frac{(v_{rel})^2}{220}\sin 45^\circ$$

$$\alpha = \frac{a_{rel,t} + \frac{(v_{rel})^2}{220} - 2\omega v_{rel}}{2(220)}$$

sub. ω, v_{rel}, and $a_{rel,t}$: $\boxed{\alpha = 0.0747 \ \text{rad/s}^2}$ (answer)

Note, if this were negative it would mean $\vec{\alpha}$ is in opposite direction