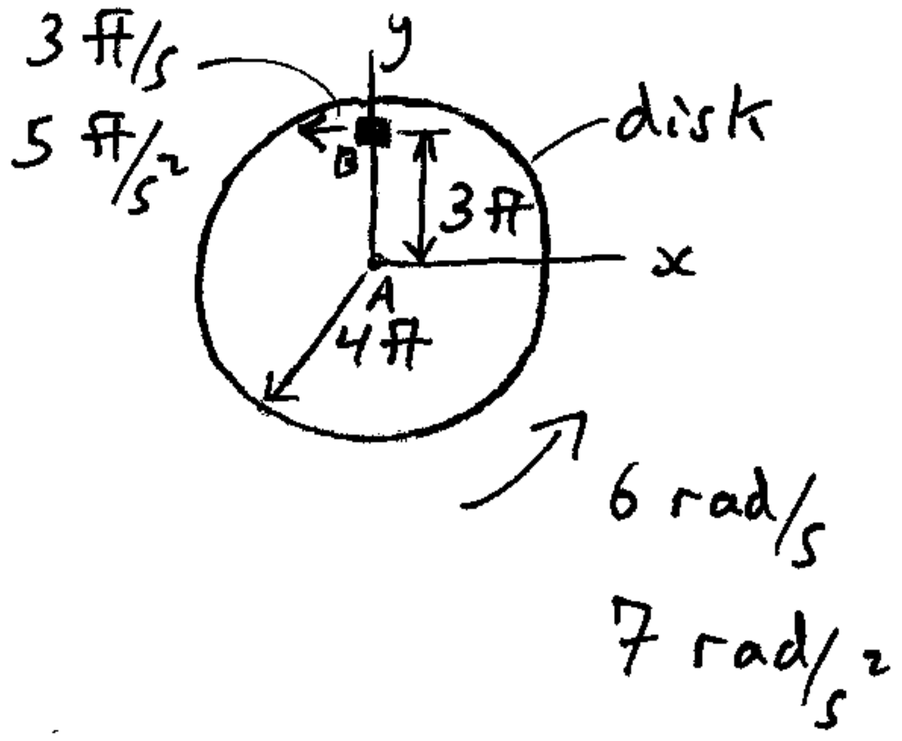
April 7, 2020

This is a 2D relative-motion analysis problem involving rotating axes (engineering mechanics).



A block B is moving along a rotating disk in the direction shown with a velocity of 3 ft/s and an acceleration of 5 ft/s² relative to the disk. The disk is also rotating in the direction shown with an angular velocity of 6 rad/s and an angular acceleration of 7 rad/s². Determine the velocity and acceleration of the block.

Solution:

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A} + (\vec{v}_{B/A})_{rel} \quad (I)$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A}) + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} + (\vec{a}_{B/A})_{rel} \quad (II)$$

Solve for velocity using (I):

$\vec{v}_A = 0$ (fixed pivot point)

$\vec{\omega} = 6\hat{k}$ rad/s $\quad (\vec{v}_{B/A})_{rel} = -3\hat{i}$ ft/s

$\vec{r}_{B/A} = 3\hat{j}$

Substitute:

$$\vec{V}_B = 0 + 6\hat{k} \times 3\hat{j} + -3\hat{i}$$

$$\vec{V}_B = -18\hat{i} - 3\hat{i} = -21\hat{i} \text{ ft/s} \quad \text{(answer)}$$

Solve for acceleration using (II):

$$\vec{a}_A = 0 \text{ (fixed pivot point)}$$

$$\vec{\alpha} = 7\hat{k} \text{ rad/s}^2$$

$$(\vec{a}_{B/A})_{rel} = -5\hat{i} \text{ ft/s}^2$$

Substitute:

$$\vec{a}_B = 0 + 7\hat{k} \times 3\hat{j} + 6\hat{k} \times (6\hat{k} \times 3\hat{j}) + 2(6\hat{k}) \times (-3\hat{i})$$

$$-5\hat{i}$$

$$\vec{a}_B = -21\hat{i} + 6\hat{k} \times (-18\hat{i}) - 36\hat{j} - 5\hat{i}$$

$$\vec{a}_B = -21\hat{i} - 108\hat{j} - 36\hat{j} - 5\hat{i}$$

$$\vec{a}_B = -26\hat{i} - 144\hat{j} \text{ ft/s}^2 \quad \text{(answer)}$$