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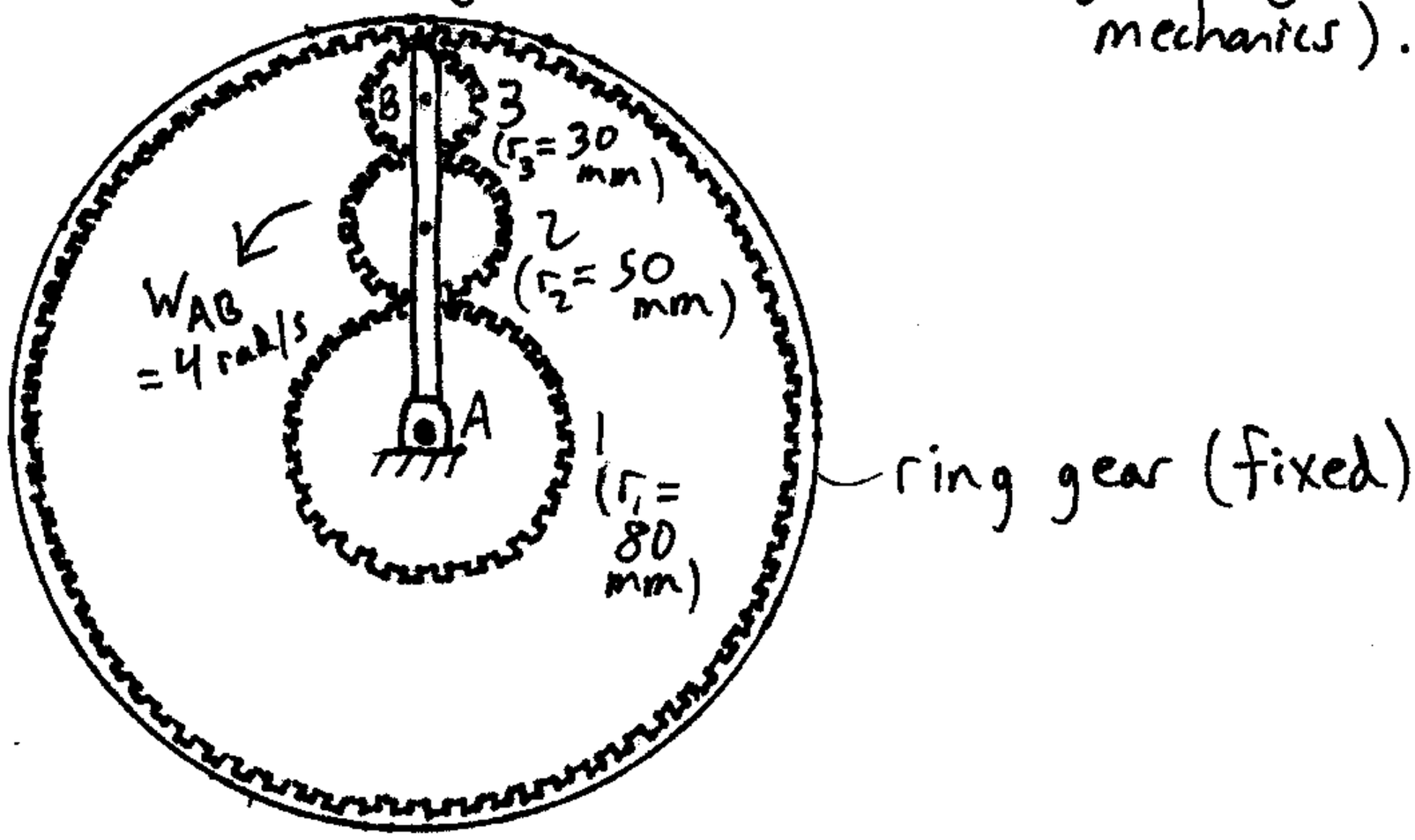
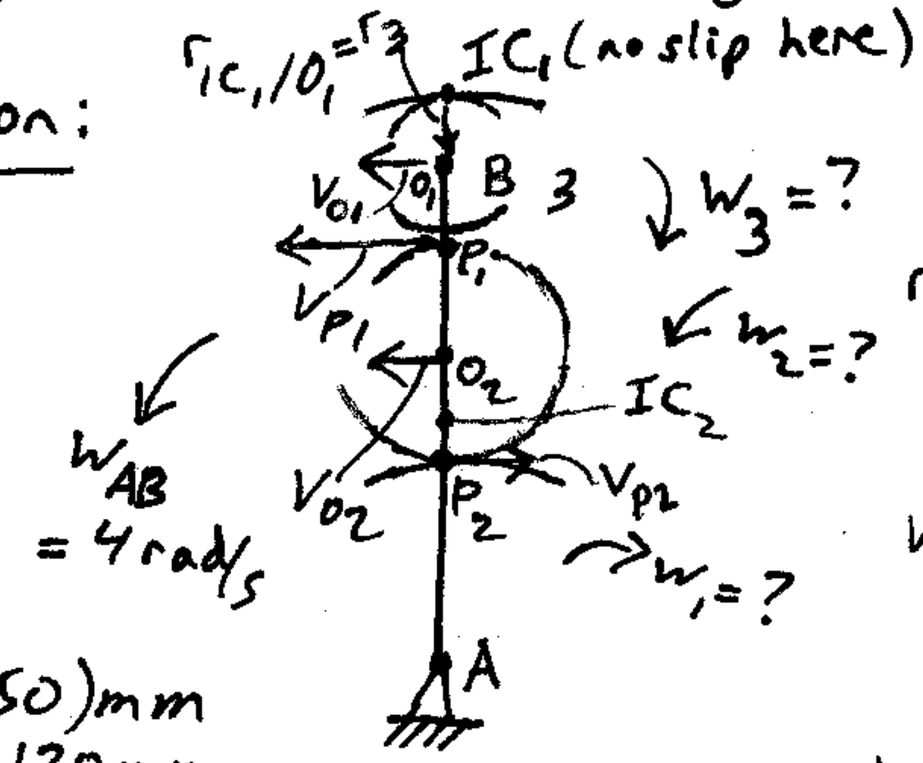
April 3, 2020

This is a problem involving instant center (engineering mechanics).



In the gear arrangement shown, the link AB has an angular velocity of $w_{AB} = 4\ rad/s$. Determine the angular velocities of gears 1, 2, and 3.

Solution:



$r_{A/O_1} = (80 + 50 + 50 + 30)\,mm = 210\ mm$

$V_{O_1} = r_{A/O_1} w_{AB} = (210)(4) = 840\ mm/s$

$w_3 = \dfrac{V_{O_1}}{r_3} = \dfrac{840\ mm/s}{30\ mm}$

$\Rightarrow w_3 = 28\ rad/s$ (answer) ↓

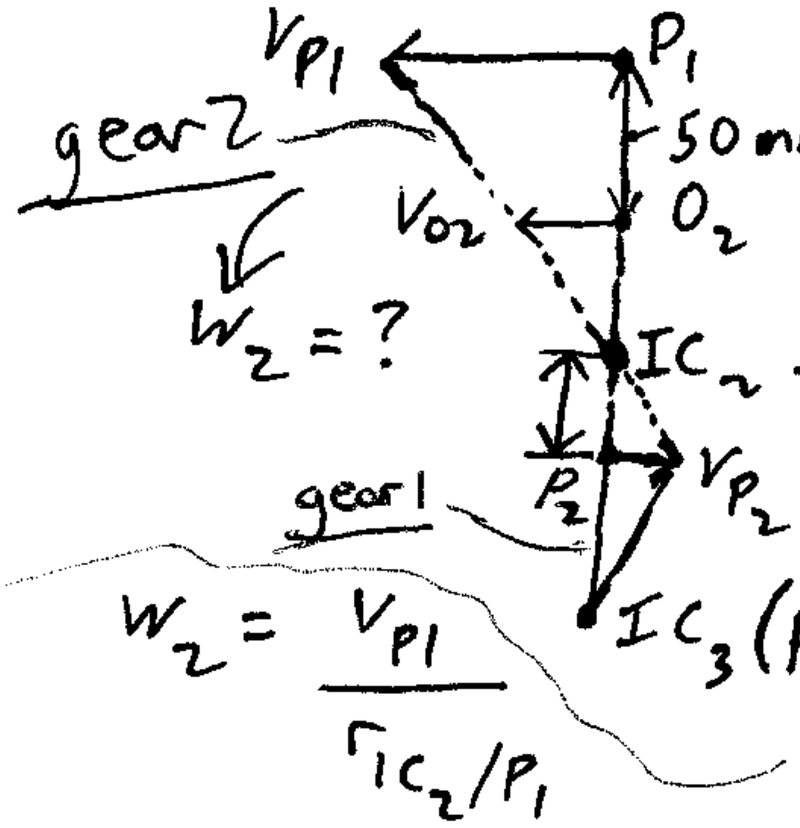
$r_{A/O_2} = (80 + 50)\,mm = 130\ mm$

$V_{P_1} = (2r_3) w_3$

$V_{P_1} = (60)(28) = 1680\ mm/s$

$V_{O_2} = r_{A/O_2} w_{AB}$

$V_{O_2} = (130)(4) = 520\ mm/s$



$$\omega_2 = \frac{v_{P1}}{r_{IC_2/P_1}}$$

$$\omega_2 = \frac{1680 \text{ mm/s}}{72.414 \text{ mm}}$$

$$\Rightarrow \omega_2 = 23.2 \text{ rad/s} \circlearrowleft \text{ (answer)}$$

By similar triangles,

$$\frac{v_{P1}}{r_{IC_2/P_1}} = \frac{v_{P_2}}{r_{P_2/IC_2}}$$

By similar triangles,

$$\frac{v_{P_1}}{r_{IC_2/P_1}} = \frac{v_{P_1} - v_{O2}}{r_{O_2/P_1}}$$

Substitute known values and solve for r_{IC_2/P_1}:

$$\frac{1680}{r_{IC_2/P_1}} = \frac{1680 - 520}{50}$$

$$r_{IC_2/P_1} = 72.414 \text{ mm}$$

By geometry,

$$r_{IC_3/P_2} = r_1$$

$$r_{P_2/IC_2} = r_2 - (72.414 - 50) \quad (= r_{IC_2/O_2})$$

$$r_{P_2/IC_2} = 50 - (72.414 - 50)$$

$$r_{P_2/IC_2} = 27.586 \text{ mm}$$

Substitute known values and solve for v_{P_2}:

$$\frac{1680}{72.414} = \frac{v_{P_2}}{27.586}, \quad v_{P_2} = 640 \text{ mm/s}$$

$$\omega_1 = \frac{v_{P_2}}{r_{IC_3/P_2}} = \frac{v_{P_2}}{r_1} = \frac{640 \text{ mm/s}}{80 \text{ mm}}$$

$$\Rightarrow \omega_1 = 8 \text{ rad/s} \circlearrowright \text{ (answer)}$$