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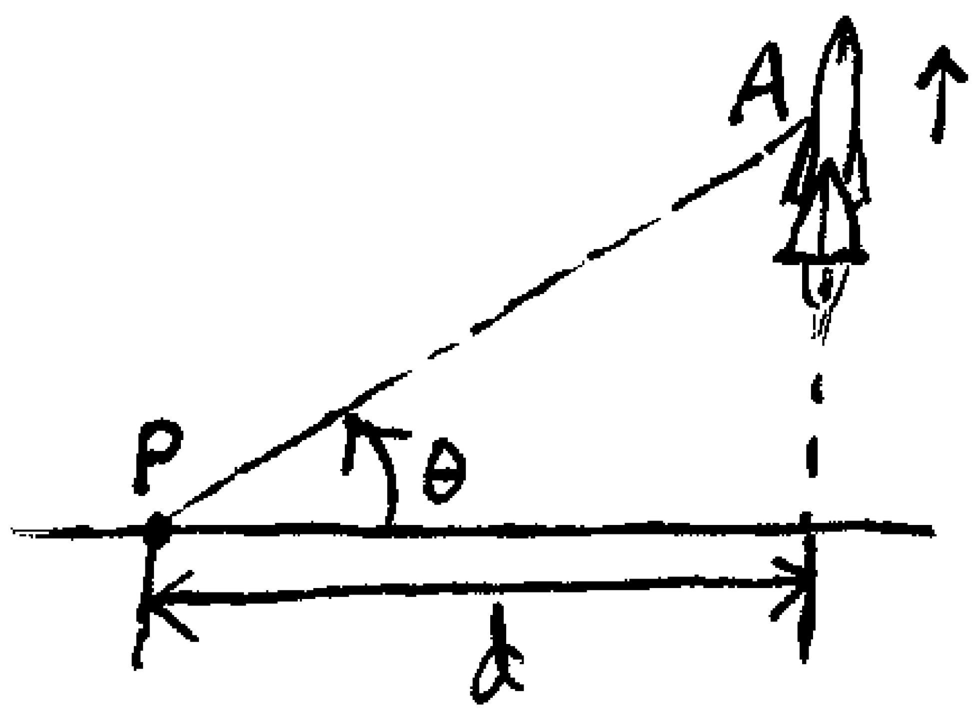
This is a curvilinear motion problem involving radial and transverse components (engineering mechanics).

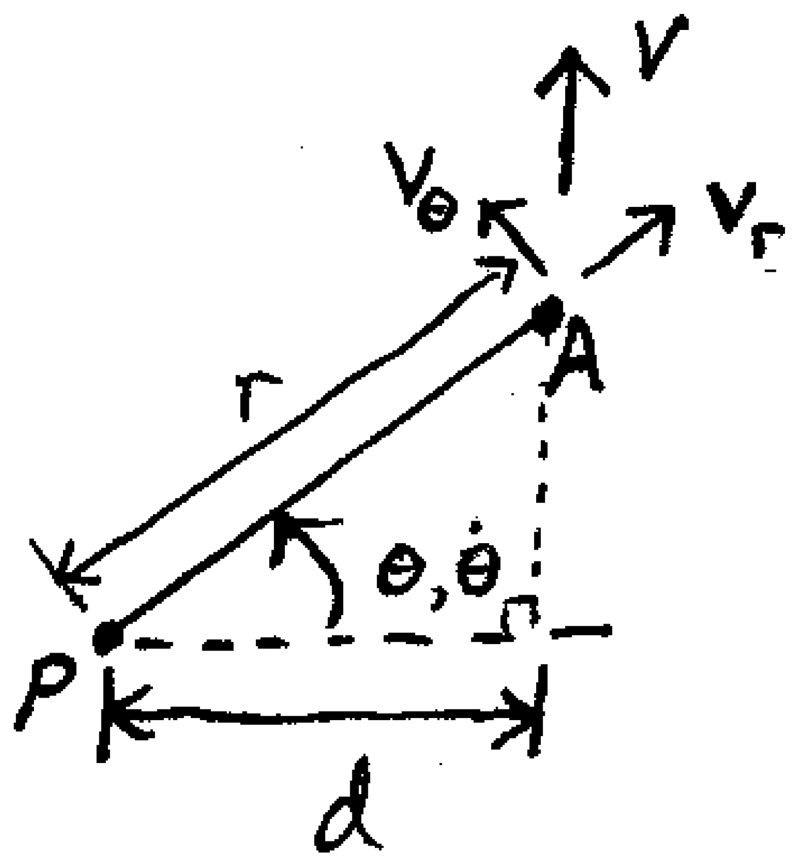


A rocket is launched vertically upward and at a given instant makes an angle of θ with the ground, as measured from point P. If the speed of the rocket is v at the instant shown, determine the speed of the rocket in terms of d, θ, and $\dot{\theta}$.

Solution:

$v_r = \dot{r}$

$v_\theta = r\dot{\theta}$



By trigonometry, $r = \dfrac{d}{\cos\theta}$

$$\dot{r} = \frac{d\sin\theta}{\cos^2\theta} \cdot \dot{\theta}$$

The resultant speed of the rocket (v) is vertically upward. This means that:

$$\frac{V_r}{V_\theta} = \tan\theta \quad (1)$$

Now,

$$v_r = \frac{d \sin\theta \cdot \dot{\theta}}{\cos^2\theta}$$

$$v_\theta = \frac{d\dot{\theta}}{\cos\theta}$$

Next, $$\frac{v_r}{V_\theta} = \frac{\frac{d \sin\theta \cdot \dot{\theta}}{\cos^2\theta}}{\frac{d\dot{\theta}}{\cos\theta}} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

So, equation (1) is automatically satisfied.

Lastly, solve for v:

$$v\cos\theta = v_\theta \quad \text{(by geometry)}$$

$$v = \frac{v_\theta}{\cos\theta} = \frac{d\dot{\theta}}{\cos^2\theta} \quad \text{(answer)}$$

Alternatively, you can solve for v using this equation instead:

$$v\sin\theta = v_r \quad \text{(by geometry)}$$

$$V = \frac{d \sin\theta \cdot \dot{\theta}}{\cos^2\theta \sin\theta} = \frac{d\dot{\theta}}{\cos^2\theta} \quad \text{(same answer)}$$