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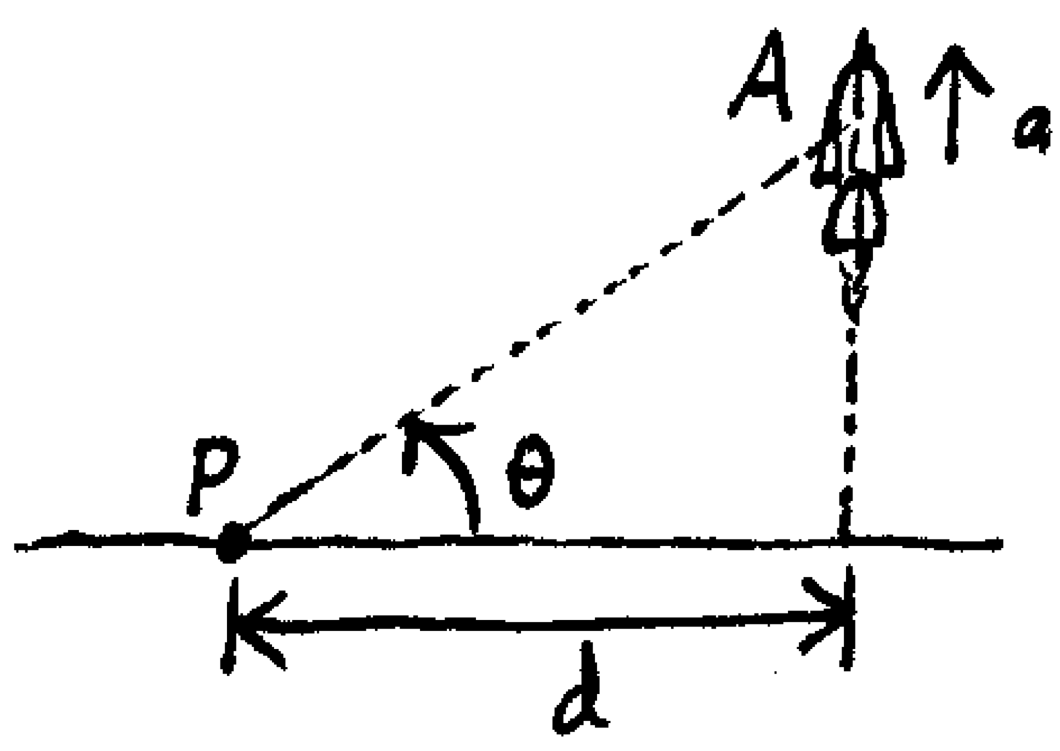
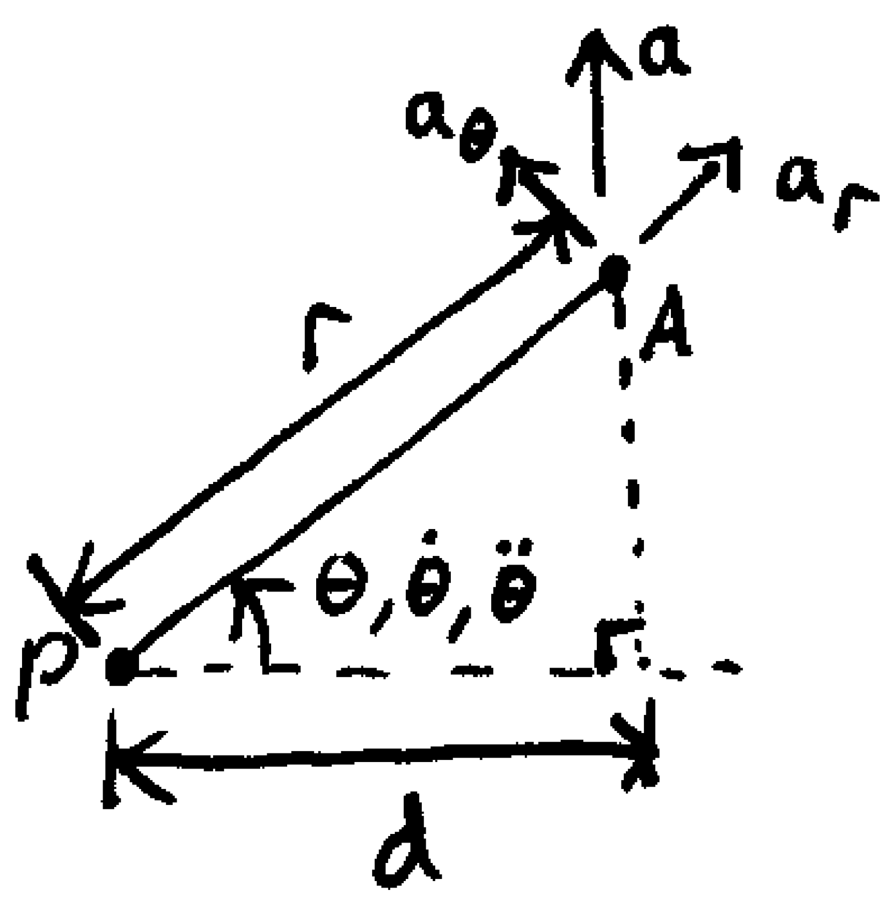
This is a curvilinear motion problem involving radial and transverse components (engineering mechanics).



A rocket is launched vertically upward and at a given instant makes an angle of θ with the ground, as measured from point P. If the acceleration of the rocket is a at the instant shown, determine the acceleration of the rocket in terms of d, θ, $\dot{\theta}$, and $\ddot{\theta}$.

Solution:



$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

By trigonometry, $r = \dfrac{d}{\cos\theta}$

$$\dot{r} = \frac{d\sin\theta}{\cos^2\theta}\cdot\dot{\theta}$$

Now, $a_\theta = \frac{d\ddot{\theta}}{\cos\theta} + \frac{2d\sin\theta}{\cos^2\theta} \cdot \dot{\theta}^2$

By geometry,

$$a\cos\theta = a_\theta$$

$$a = \frac{d\ddot{\theta}}{\cos^2\theta} + 2\frac{d\sin\theta}{\cos^3\theta} \cdot \dot{\theta}^2 \quad \text{(answer)}$$

Note, that if we were to solve for a using this equation:

$$a\sin\theta = a_r$$

we would get the same answer!

→ but it is easier to solve for a using the equation $a\cos\theta = a_\theta$ since we only have to determine $\dot{r}$. If we were to use the equation: $a\sin\theta = a_r$, we would have to determine $\ddot{r}$, which is more work!