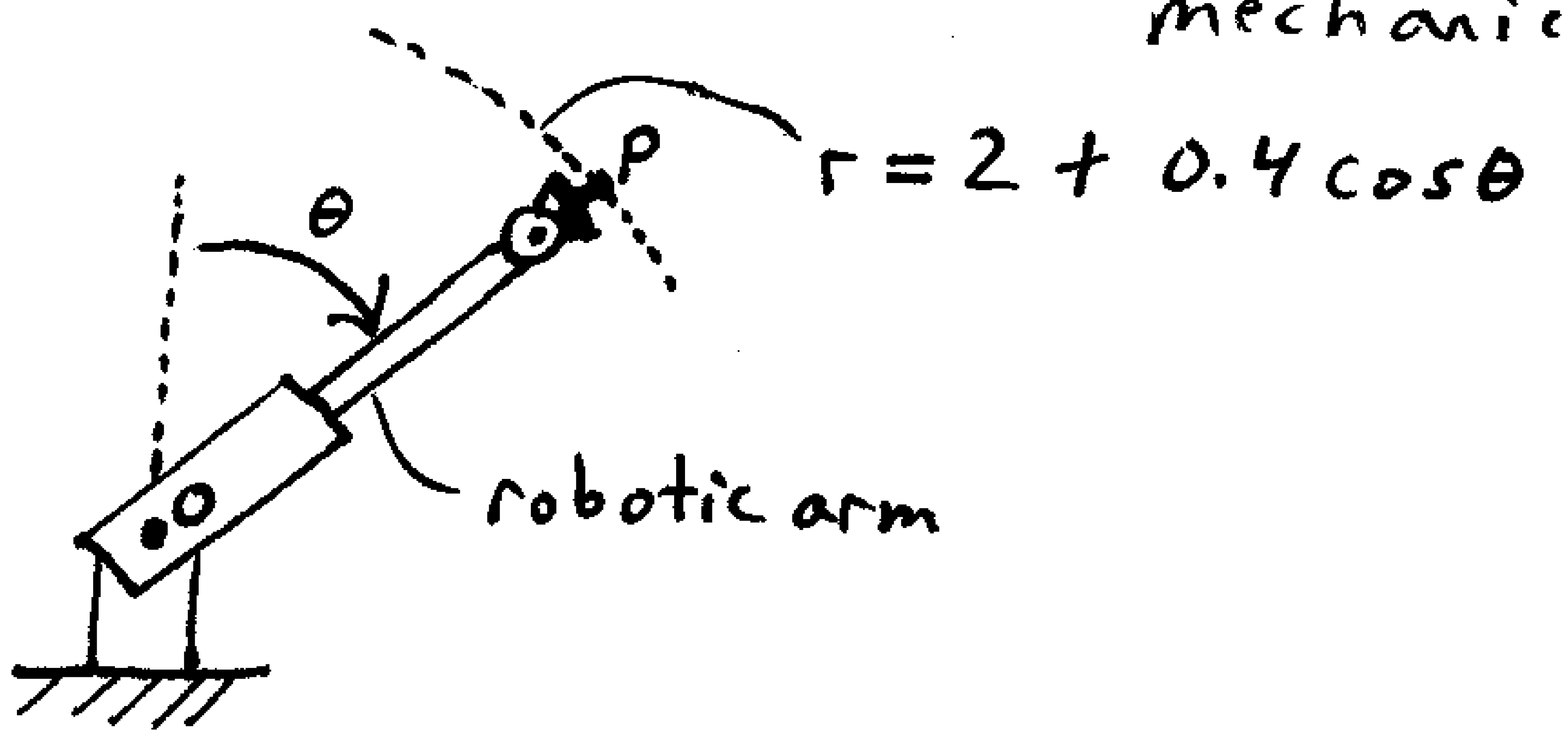
This is a curvilinear motion problem involving radial and transverse components (engineering mechanics).



A robotic arm with endpoint P rotates about point O, while also changing its length such that the distance from point O to point P is given by $r = 2 + 0.4\cos\theta$ meters. What is the greatest constant angular velocity of the arm, $\dot{\theta}$, so that the maximum acceleration of P is $6\ m/s^2$. Calculate the answer over the range $-45° \le \theta \le 45°$.

Solution:

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\underbrace{\ddot{\theta}}_{0} + 2\dot{r}\dot{\theta} = 2\dot{r}\dot{\theta}$$

Substitute:

$$r = 2 + 0.4\cos\theta$$
$$\dot{r} = -0.4\sin\theta\cdot\dot{\theta}$$
$$\ddot{r} = -0.4\cos\theta\cdot\dot{\theta}^2$$

$$a_r = -0.4\cos\theta\cdot\dot{\theta}^2 - (2 + 0.4\cos\theta)\dot{\theta}^2$$
$$= -0.8\cos\theta\cdot\dot{\theta}^2 - 2\dot{\theta}^2$$
$$a_\theta = 2(-0.4\sin\theta\cdot\dot{\theta})\dot{\theta}$$
$$= -0.8\sin\theta\cdot\dot{\theta}^2$$

The magnitude of the acceleration is

$$a_p = \sqrt{a_r^2 + a_\theta^2}$$

$$a_p = \sqrt{(-0.8\cos\theta\cdot\dot{\theta}^2 - 2\dot{\theta}^2)^2 + (-0.8\sin\theta\cdot\dot{\theta}^2)^2}$$

$$a_p = \dot{\theta}^2\sqrt{(0.8\cos\theta + 2)^2 + (0.8\sin\theta)^2}$$

$$a_p = \dot{\theta}^2\sqrt{0.8^2 + 3.2\cos\theta + 4}$$

The maximum value of a_p occurs when $\cos\theta = 1$, for $\theta = 0$, which is within the range $-45^\circ \le \theta \le 45^\circ$.

At $\theta = 0$, $a_p = \dot{\theta}^2\sqrt{0.8^2 + 3.2 + 4}$

$$a_p = \dot{\theta}^2\sqrt{(7.84)}$$

$$a_p = 2.8\dot{\theta}^2$$

Now, $a_p \le 6\ m/s^2$

$$\Rightarrow 2.8\dot{\theta}^2 \le 6$$

so $\dot{\theta} \le 1.46$ rad/s

Greatest angular velocity is 1.46 rad/s

(answer)