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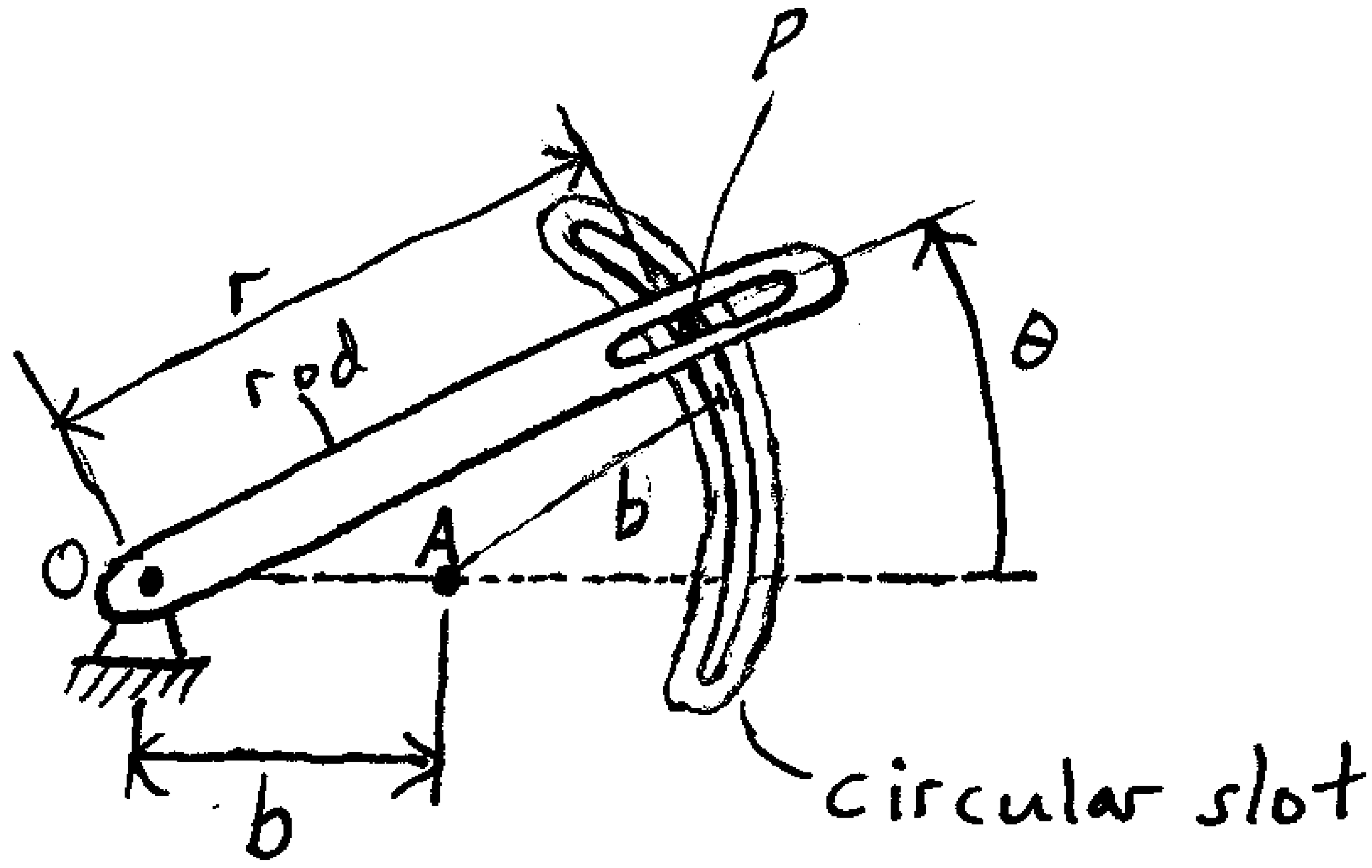
This is a curvilinear motion problem involving radial and transverse components (engineering mechanics).



A rod is rotating about a point O, while pushing a pin P along a circular slot, as shown. The pin P is also free to slide along a slot in the rod, as shown. If the rod rotates at constant rate $\dot{\theta}$, show that the acceleration of P has constant magnitude, and determine the direction of this acceleration.

Solution:

By geometry, $r = 2b\cos\theta$

$$\dot{r} = -2b\sin\theta\cdot\dot{\theta}$$

$$\ddot{r} = -2b\left(\cos\theta\cdot(\dot{\theta})^2 + \sin\theta\cdot\overset{\nearrow 0,\ \text{since } \dot{\theta} \text{ is constant}}{\ddot{\theta}}\right)$$

$$\ddot{r} = -2b\cos\theta\cdot(\dot{\theta})^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2b\cos\theta\cdot(\dot{\theta})^2 - 2b\cos\theta\cdot(\dot{\theta})^2$$
$$= -4b\cos\theta\cdot(\dot{\theta})^2$$

$$a_\theta = r\underset{\downarrow \atop 0}{\ddot{\theta}} + 2\dot{r}\dot{\theta} = 2(-2b\sin\theta\cdot\dot{\theta})\cdot\dot{\theta} = -4b\sin\theta\cdot(\dot{\theta})^2$$

The magnitude of the acceleration is:

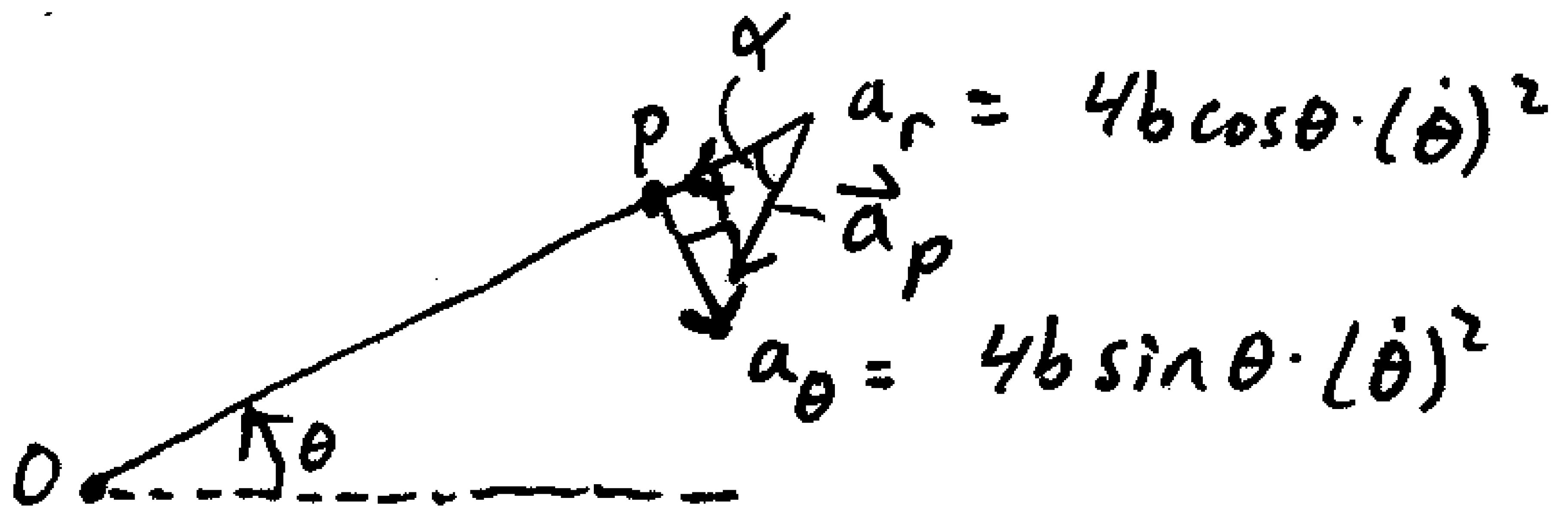
$$|\vec{a}_p| = \sqrt{a_r^2 + a_\theta^2} = \sqrt{\left(-4b\cos\theta\cdot(\dot{\theta})^2\right)^2 + \left(-4b\sin\theta\cdot(\dot{\theta})^2\right)^2}$$

$$= 4b(\dot{\theta})^2\sqrt{\cos^2\theta + \sin^2\theta} = 4b(\dot{\theta})^2\sqrt{(1)}$$

$$= 4b(\dot{\theta})^2 \quad \text{(constant)}$$

(answer)

Now, find the direction of this acceleration:



$$\alpha = \tan^{-1}\left(\frac{a_\theta}{a_r}\right) = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$= \tan^{-1}(\tan\theta)$$

$$= \theta$$

Therefore $\alpha = \theta$

This means that the acceleration of the pin has direction pointing towards point A, as shown in the figure, which is the center point of the circular slot (answer).