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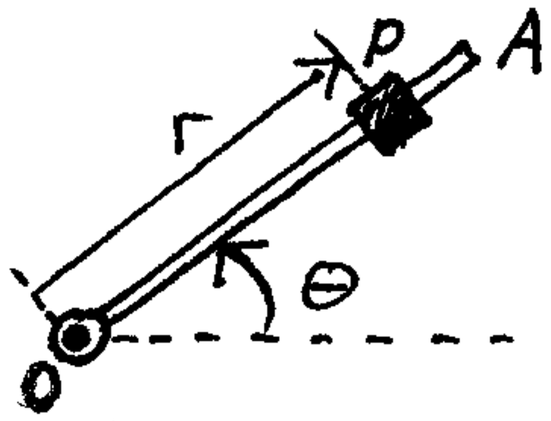
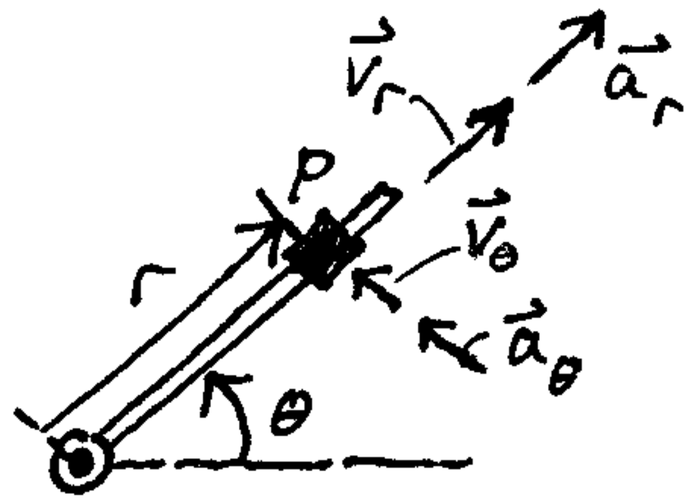
This is a curvilinear motion problem involving radial and transverse components (engineering mechanics).



The rod OA is rotating in a horizontal plane about point O such that $\theta = 2t^3$, where t is in seconds. At the same time the collar P is sliding outward along the rod such that $r = 40t^2$. Determine the magnitude of the velocity and acceleration of the collar when $t = 1.5$ s. Distances are measured in mm, and angles in radians.

Solution:



$v_r = \dot{r}$ (magnitude of the velocity vector $\vec{v}_r$)

$v_\theta = r\dot{\theta}$ (magnitude of the velocity vector $\vec{v}_\theta$)

$a_r = \ddot{r} - r\dot{\theta}^2$ (magnitude of the acceleration vector $\vec{a}_r$)

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ (magnitude of the acceleration vector $\vec{a}_\theta$)

At $t = 1.5s$,

$r = 40(1.5)^2 = 90\ mm$

$\dot{r} = 80t\Big|_{t=1.5s} = 120\ mm/s$

$\ddot{r} = 80\ mm/s^2$

$\dot{\theta} = 6t^2\Big|_{t=1.5s} = 13.5\ rad/s$

$\ddot{\theta} = 12t\Big|_{t=1.5s} = 18\ rad/s^2$

Substitute:

$v_r = 120\ mm/s$

$v_\theta = (90)(13.5) = 1215\ mm/s$

$a_r = 80 - (90)(13.5)^2 = -16322.5\ mm/s^2$

$a_\theta = (90)(18) + 2(120)(13.5) = 4860\ mm/s^2$

The magnitude of the velocity is: $|\vec{V}| = \sqrt{v_\theta^2 + v_r^2}$
$= 1221\ mm/s$ (ans.)

The magnitude of the acceleration is: $|\vec{a}| = \sqrt{a_\theta^2 + a_r^2}$
$= 17031\ mm/s^2$
(ans.)