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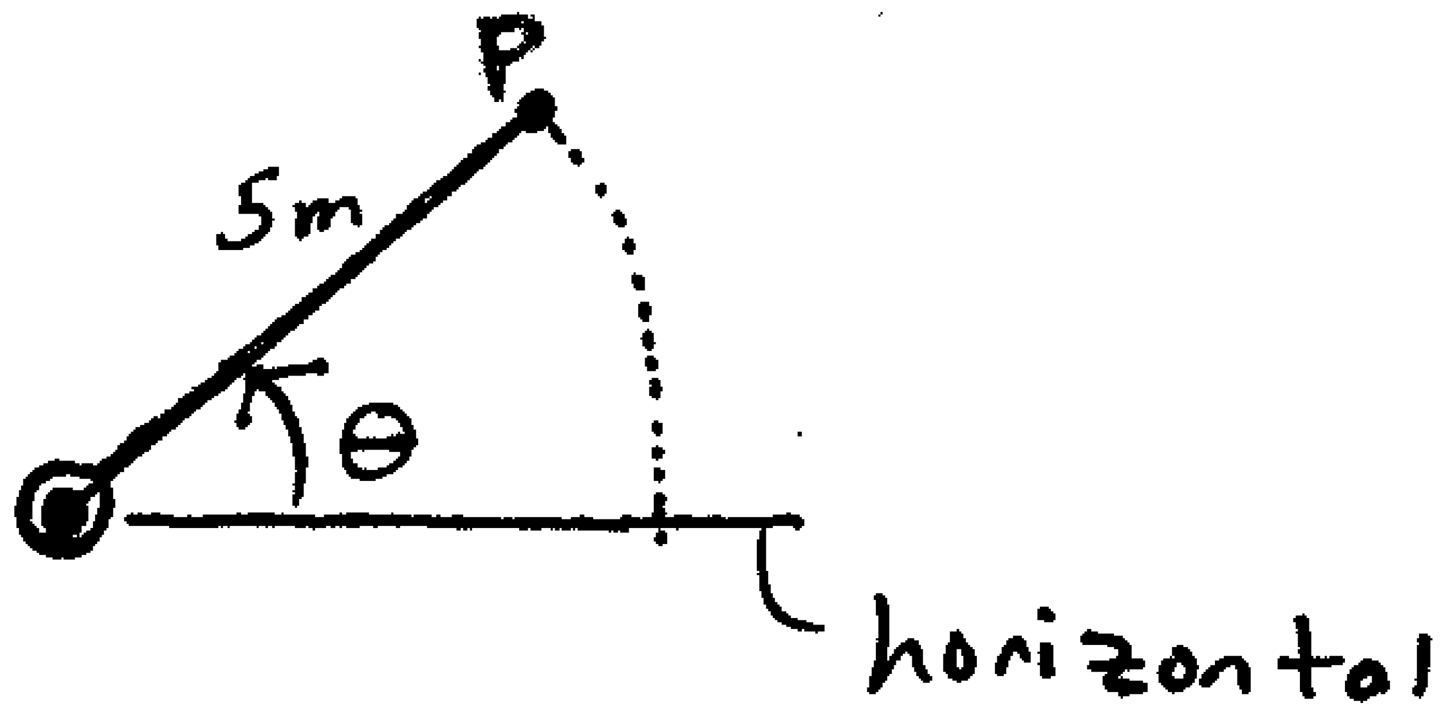
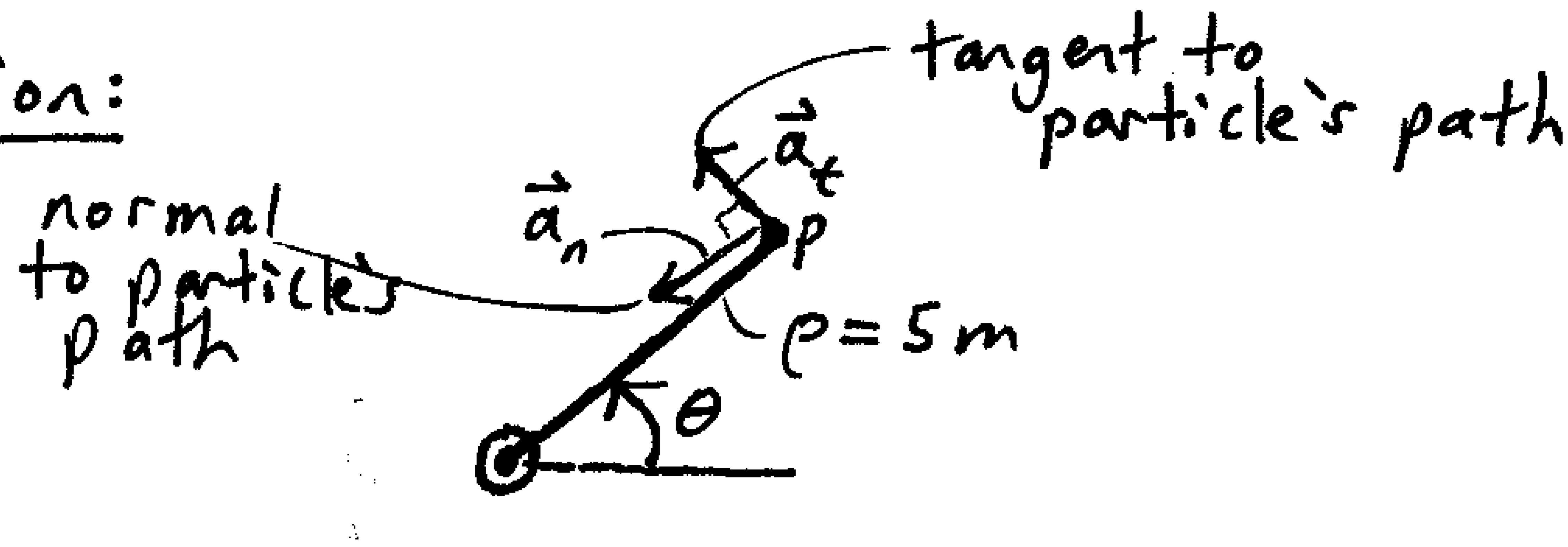
March 16, 2020

This is a curvilinear motion problem involving normal and tangential components (engineering mechanics).



A particle P is traveling along a circular arc of radius 5 m. The speed of the particle is given by $v = 0.5e^{t}\sin(2t)$ m/s, where t is in seconds and the argument for the sine is in radians. Determine the magnitude and direction of the acceleration of P when $t = 2.5$ s. The particle starts from rest when $\theta = 0°$.

Solution:



Note that $\vec{a}_t$ can be pointing in the opposite direction – the math will still work out.

$v = \frac{ds}{dt}$, where s is the travel distance, in meters.

Now, $\frac{ds}{dt} = 0.5 e^t \sin(2t)$

Integrate: $s = \frac{1}{10} e^t \left(\sin(2t) - 2\cos(2t) \right) + C$

C is a constant of integration

For convenience, at $t=0$, $s=0$ (and $\theta = 0°$ as given in problem)

Solve for C when $s=0$ and $t=0$:

$$0 = \frac{1}{10} e^0 \left(\sin(0) - 2\cos(0) \right) + C$$

$$0 = -\frac{1}{5} + C$$

$$C = \frac{1}{5}$$

Therefore, $s = \frac{1}{10} e^t \left(\sin(2t) - 2\cos(2t) \right) + \frac{1}{5}$

when $t = 2.5$ seconds,

$$s = \frac{1}{10} e^{2.5} \left(\sin(5) - 2\cos(5) \right) + \frac{1}{5} = -1.66 \text{ m}$$

Since $s < 0$ this means that $\theta < 0$

and $\theta = \frac{s}{\rho} = \frac{-1.66\text{ m}}{5\text{ m}} = -0.3319$ radians

The magnitude of the normal acceleration is:

$a_n = \frac{v^2}{\rho}$ (a_n is the magnitude of the acceleration vector $\vec{a}_n$, normal to the path at any given instant)

The magnitude of the tangential acceleration is:

$a_t = \frac{dv}{dt}$ (magnitude of the acceleration vector $\vec{a}_t$, tangent to the path at any given instant)

At $t = 2.5\,s$, $v = 0.5e^{2.5}\sin(5) = -5.841\ m/s$

Now, $a_t = \frac{dv}{dt} = 0.5(e^t \sin(2t) + 2e^t \cos(2t))$

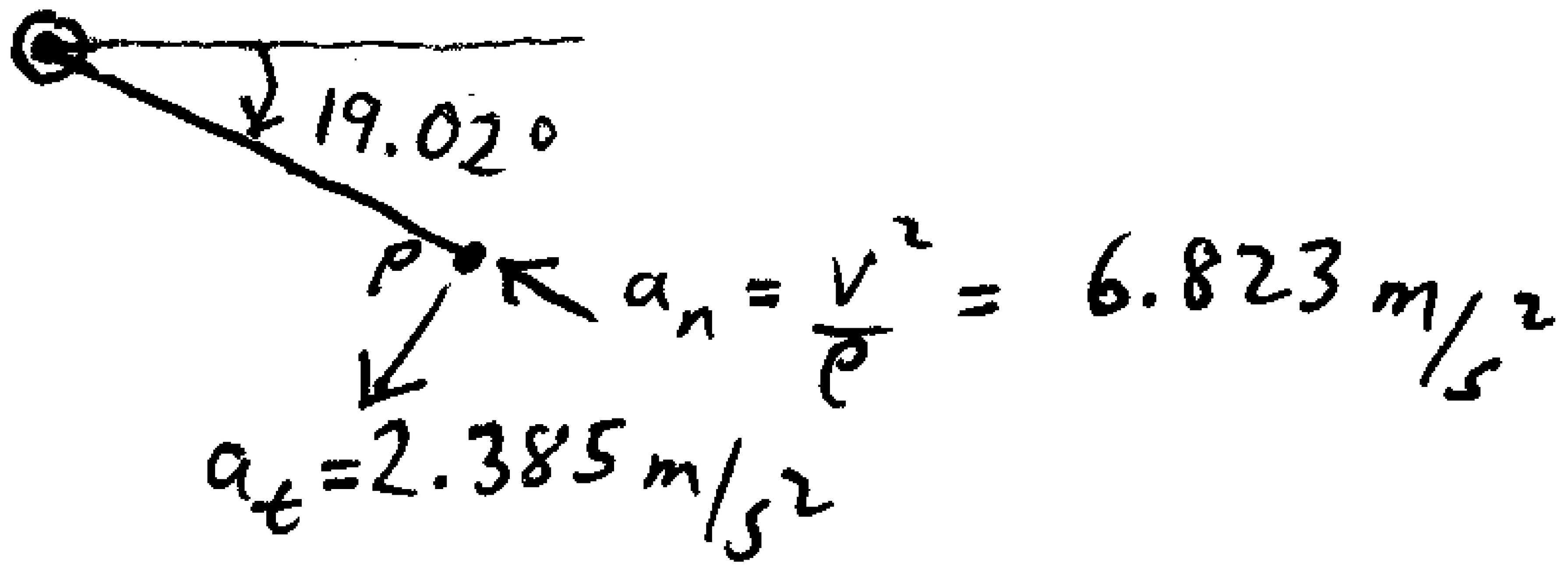
At $t = 2.5\,s$, $a_t = 0.5(e^{2.5}\sin(5) + 2e^{2.5}\cos(5))$

$a_t = -2.385\ m/s^2$

The negative sign means that the tangential acceleration of P is in a direction opposite to that shown in the diagram.

From before, $\theta = -0.3319$ radians
$= -19.02°$

(negative sign means θ is below the horizontal, as shown)



The magnitude of the acceleration of P is:

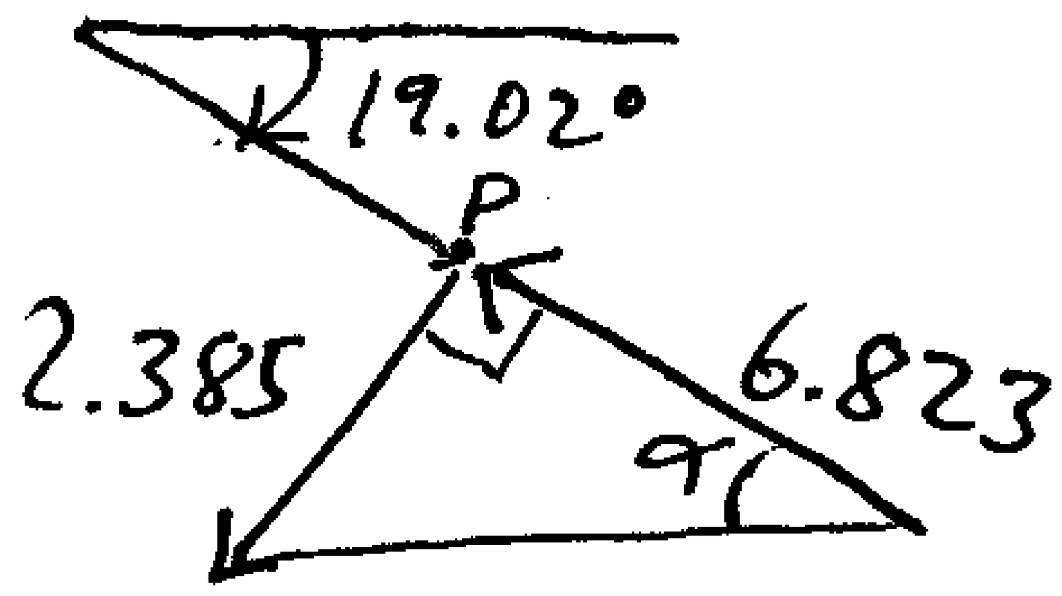
$$|\vec{a}_p| = \sqrt{a_n^2 + a_t^2}$$

where $\vec{a}_p$ is the acceleration vector of P at $t = 2.5$ s.

$$= \sqrt{(6.823)^2 + (2.385)^2}$$

$$= 7.23 \text{ m/s}^2 \quad \text{(answer)}$$

Now, find the direction of the acceleration of P:



$$\theta = \tan^{-1}\left(\frac{2.385}{6.823}\right) = 19.27°$$

This means the acceleration of P has a direction that is $19.27° - 19.02° = 0.25°$ below the horizontal.
(answer)