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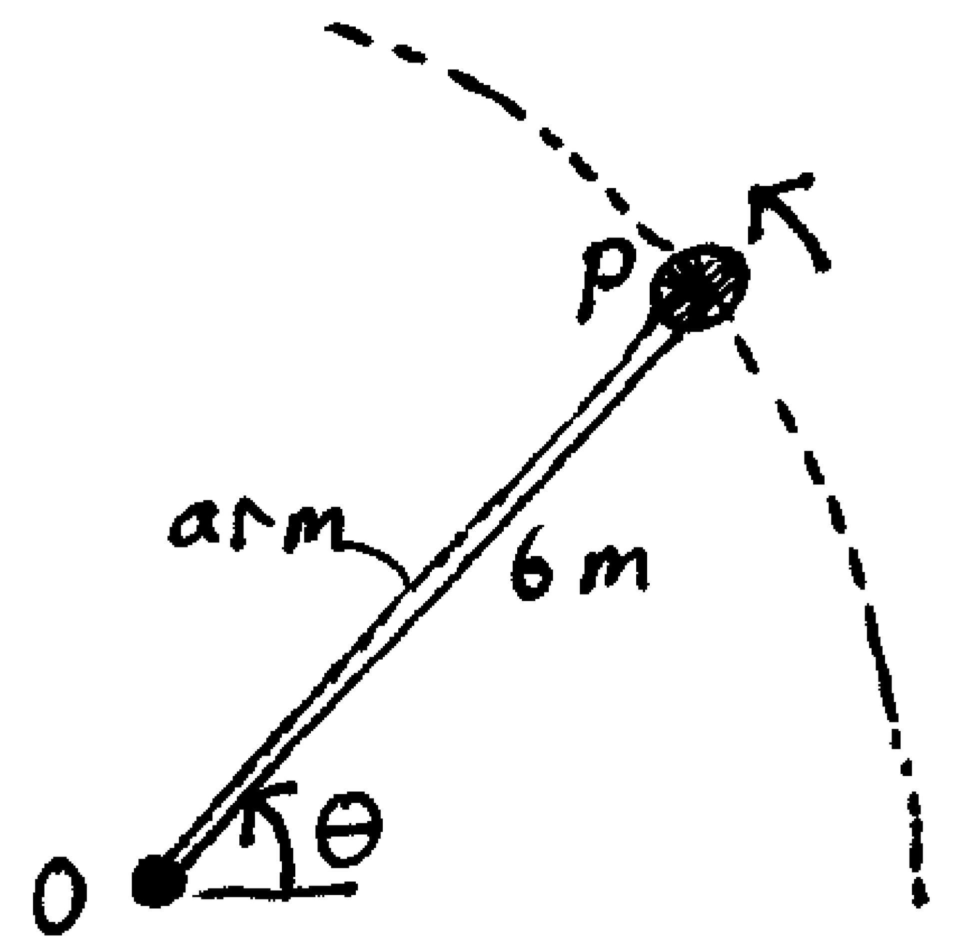
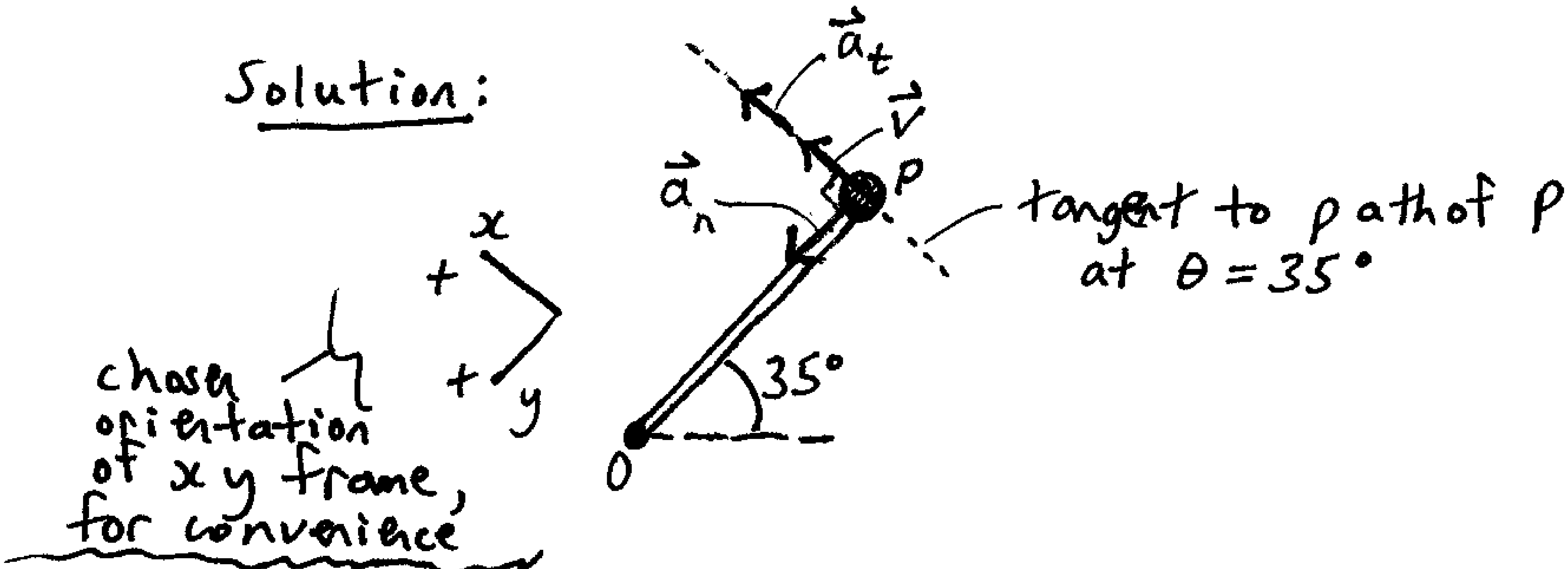
This is a curvilinear motion problem involving normal and tangential components (engineering mechanics).



An arm rotates about a pivot point O, and its end P speed increases according to the equation $v = 0.6e^{t} + 2t$, where t is in seconds. The arm starts from rest at $\theta = 0$, and the length of the arm is 6 m. Determine the normal and tangential components of the velocity and acceleration of the end P when $\theta = 35°$.

Solution:



The magnitude of the normal acceleration is:

$a_n = \frac{v^2}{\rho}$ (a_n is the magnitude of the acceleration vector $\vec{a}_n$, normal to the path of P, at the instant shown)

The magnitude of the tangential acceleration is:

$a_t = \frac{dv}{dt} = \dot{v}$ (magnitude of the acceleration vector $\vec{a}_t$, tangent to the path of P, at the instant shown)

$\dot{v} = 0.6e^t + 2t$ (v is the speed of P, which is the magnitude of the velocity vector $\vec{v}$, tangent to the path of P, at the instant shown)

Since $\frac{dv}{dt} = 0.6e^t + 2t$

Integrate: $v = 0.6e^t + t^2 + C_1$, where C_1 is a constant of integration

Integrate: $s = 0.6e^t + \frac{t^3}{3} + C_1 t + C_2$, where C_2 is a constant of integration, and s is the distance travelled by P, as measured from when $\theta = 0$

At $t=0$, $v=0$, so $C_1 = -0.6$
(starts from rest)

At $t=0$, $s=0$, so $C_2 = -0.6$
(zero travel distance initially, by definition, and for convenience)

Therefore, $v = 0.6e^t + t^2 - 0.6$
$s = 0.6e^t + \frac{t^3}{3} - 0.6t - 0.6$

When $\theta = 35°$, the travel distance of P is:

$s = R\theta$, where R is the radius of the arc travelled by P and θ is the angle, in radians.

Now, $R = 6\,m$, and $\theta = \frac{\pi}{180} \times 35 = 0.611$ radians

Therefore, $s = 6 \times 0.611 = 3.665\,m$

Next, solve for t in the equation:

$$s = 3.665 = 0.6e^t + \frac{t^3}{3} - 0.6t - 0.6$$

Solve for t with a computer or programmable calculator.

$t = 1.763\,s$

Next, solve for v at $t = 1.763\,s$.

$$v = 0.6e^{(1.763)} + (1.763)^2 - 0.6$$
$$v = 6.0\,m/s$$

The velocity of P is in a direction tangent to the path of P at $\theta = 35°$, and has a magnitude of $6.0\,m/s$. (answer)

} there is no normal component → velocity direction always tangent to travel path

The acceleration of P has a tangential component ($\vec{a}_t$) and a normal component ($\vec{a}_n$) to the path of P at $\theta = 35°$, when $t = 1.763\,s$.

magnitudes {

$$a_t = 0.6e^{(1.763)} + 2(1.763) = 7.0\,m/s^2 \text{ (answer)}$$

$$a_n = \frac{v^2}{\rho} = \frac{(6.0)^2}{6} = 6.0\,m/s^2 \text{ (answer)}$$