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This is a curvilinear motion problem involving normal and tangential components (engineering mechanics).

The position of a particle is defined by $\vec{r} = (t^2+t)\hat{i} + (2e^t \sin 2t)\hat{j}$ ft, where t is in seconds and the argument for sine is in radians. When $t = 3$ s, determine the speed of the particle and its normal and tangential components of acceleration.

Solution:

The x-component of the particle's speed is:

$$v_x = \frac{d}{dt}(t^2+t) = 2t+1$$

The y-component of the particle's speed is:

$$v_y = \frac{d}{dt}(2e^t \sin 2t)$$

$$v_y = 2(e^t \sin 2t + 2e^t \cos 2t)$$

At $t = 3$ s,

$$v_x = 2(3)+1 = 7 \text{ ft/s}$$

$$v_y = 2\left(e^3 \sin[2(3)] + 2e^3 \cos[2(3)]\right) = 65.91 \text{ ft/s}$$

The speed is $\sqrt{(v_x)^2+(v_y)^2} = 66.3$ ft/s (answer)

The slope of the tangent is:

$$\frac{dy}{dx} = ?$$

Since we don't have y given in terms of x we need to apply the chain rule of Calculus:

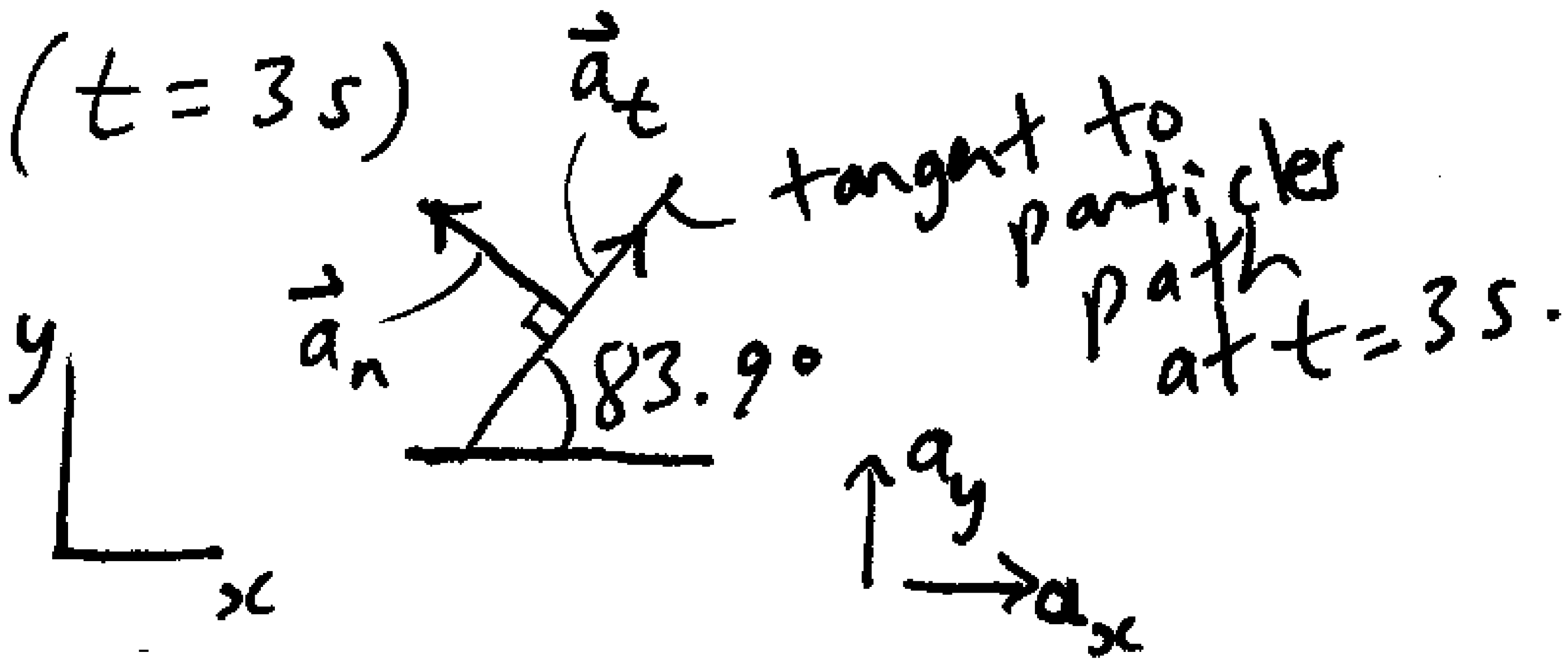
$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy/dt}{\frac{dx}{dt}} = \frac{V_y}{V_x}$$

At $t = 3s$, $\frac{dy}{dx} = \frac{65.91}{7} = 9.41 = \tan\theta$,

where θ is the angle between the x-axis and the tangent to the curve $y = f(x)$ at $t = 3$ s.

Solve for $\theta = 83.9°$

(t = 3s)



The x-component of the particle's acceleration is:

$$a_x = \frac{dV_x}{dt} = 2 \text{ ft}/s^2$$

The y-component of the particle's acceleration is:

$$a_y = \frac{dv_y}{dt} = 2(e^t \sin 2t + 2e^t \cos 2t) + 4(e^t \cos 2t - 2e^t \sin 2t)$$

At $t = 3s$,

$$a_y = 187.96 \ ft/s^2$$

The magnitude of the tangential acceleration vector $\vec{a}_t$ is:

$$a_t = a_x \cos 83.9° + a_y \sin 83.9°$$

$$a_t = 187.1 \ ft/s^2 \quad \text{(answer)}$$

The magnitude of the normal acceleration vector $\vec{a}_n$ is:

$$a_n = a_y \cos 83.9° - a_x \sin 83.9°$$

$$a_n = 17.98 \ ft/s^2 \quad \text{(answer)}$$