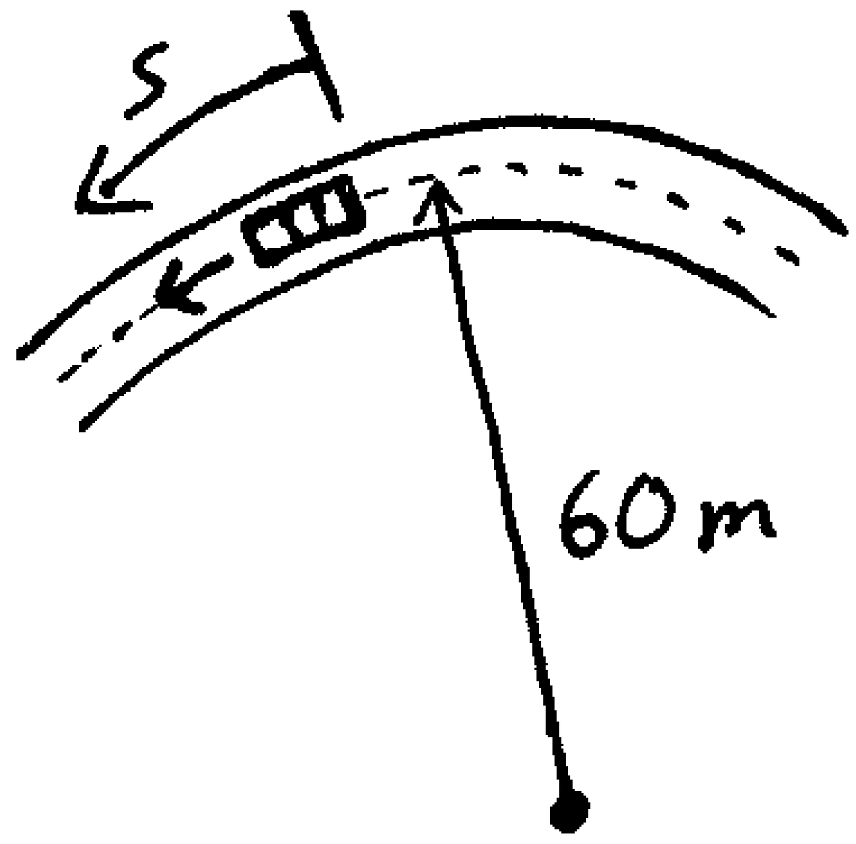
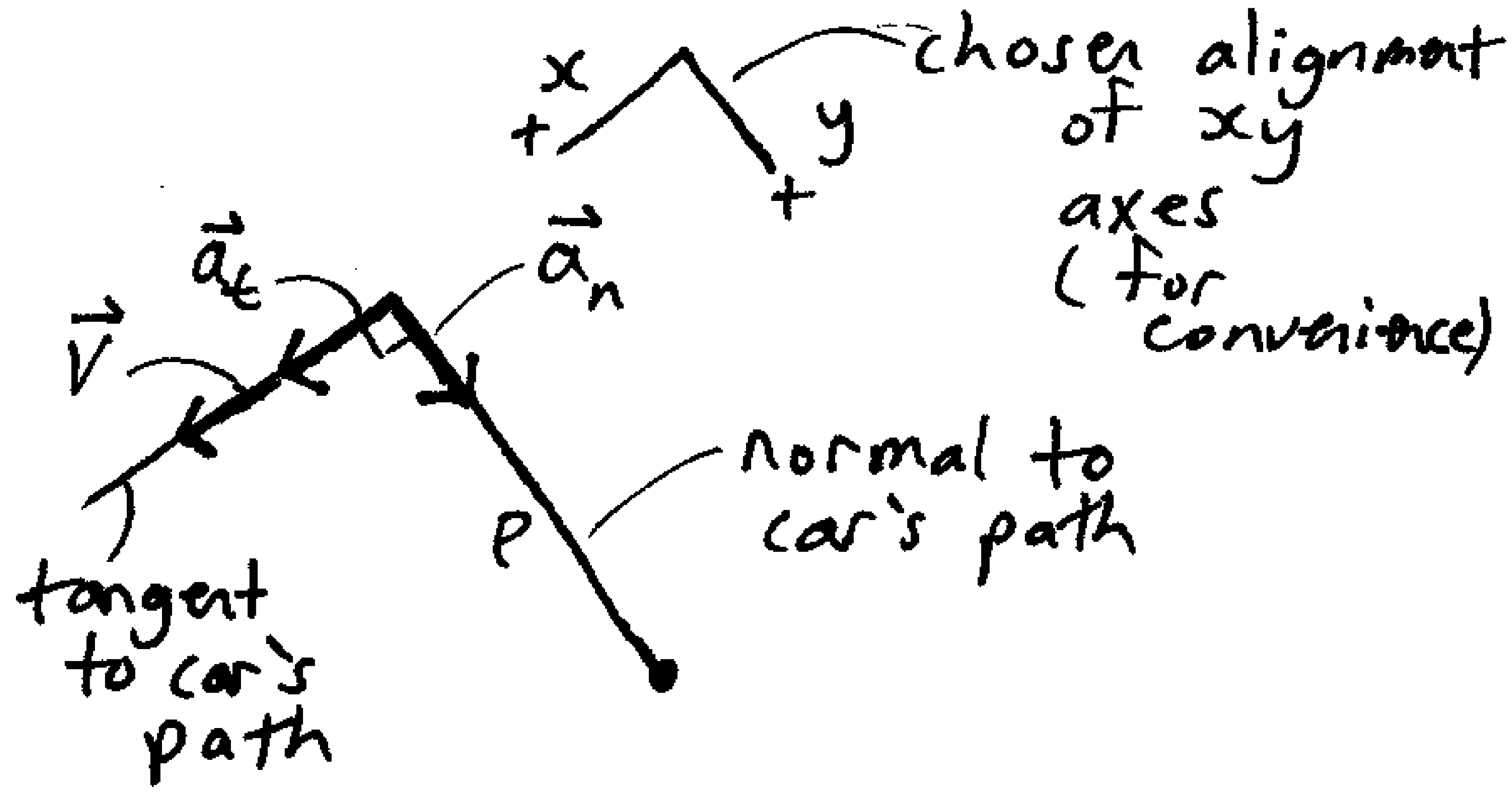
Feb. 28, 2020

This is a curvilinear motion problem involving normal and tangential components (engineering mechanics).



A car travels at a speed of 8 m/s along a circular road which has a radius of 60 m. Starting from $s = 0$, where s is the travel distance, in meters, the car increases its speed by $\dot{v} = (0.07s)$ m/s². Calculate the speed of the car and the magnitude of its acceleration when $s = 12$ m.

Solution:



The magnitude of the normal acceleration is:

$$a_n = \frac{v^2}{\rho}$$

(a_n is the magnitude of the acceleration vector $\vec{a}_n$, normal to the path at any given instant)

The magnitude of the tangential acceleration is:

$$a_t = \frac{dv}{dt} = \dot{v} = 0.07s$$ (magnitude of the acceleration vector $\vec{a}_t$, tangent to the path at any given instant)

We need to find an expression for the speed, v, as a function of s. Note that v is the speed of the car, which is the magnitude of the velocity vector $\vec{v}$, tangent to the path at any given instant.

Given, $\frac{dv}{dt} = 0.07s$

By the chain rule of Calculus,

$$\frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt}$$, and since $\frac{ds}{dt} = v$,

then $\frac{dv}{dt} = \frac{dv}{ds} \cdot v$

Now, $\frac{dv}{ds} \cdot v = 0.07s$

$$dv \cdot v = 0.07s \cdot ds$$

Integrate: $\frac{1}{2}v^2 = 0.035s^2 + C$

where C is the constant of integration

At $s = 0$, $v = 8\ m/s$

Substitute:

$$\frac{1}{2}(8)^2 = 0.035(0)^2 + C$$

$$C = 32$$

Then, $\frac{1}{2}v^2 = 0.035s^2 + 32$

$$v = \sqrt{0.07s^2 + 64}$$

When $s = 12\ m$,

$$v = \sqrt{0.07(12)^2 + 64} = 8.6\ m/s$$ (answer) (speed of the car)

and, $\dot{v} = 0.07(12) = 0.84\ m/s^2$

$$= a_t$$

The horizontal component of the resultant acceleration is:

$$a_x = a_t = 0.84\ m/s^2 \text{ (at } s = 12m)$$

The vertical component of the resultant acceleration is:

$$a_y = a_n = \frac{v^2}{\rho} = \frac{(8.6)^2}{60} = 1.23\ m/s^2 \text{ (at } s = 12m)$$

The magnitude of the resultant acceleration is: $\sqrt{(0.84)^2 + (1.23)^2} = 1.5\ m/s^2$

(answer)