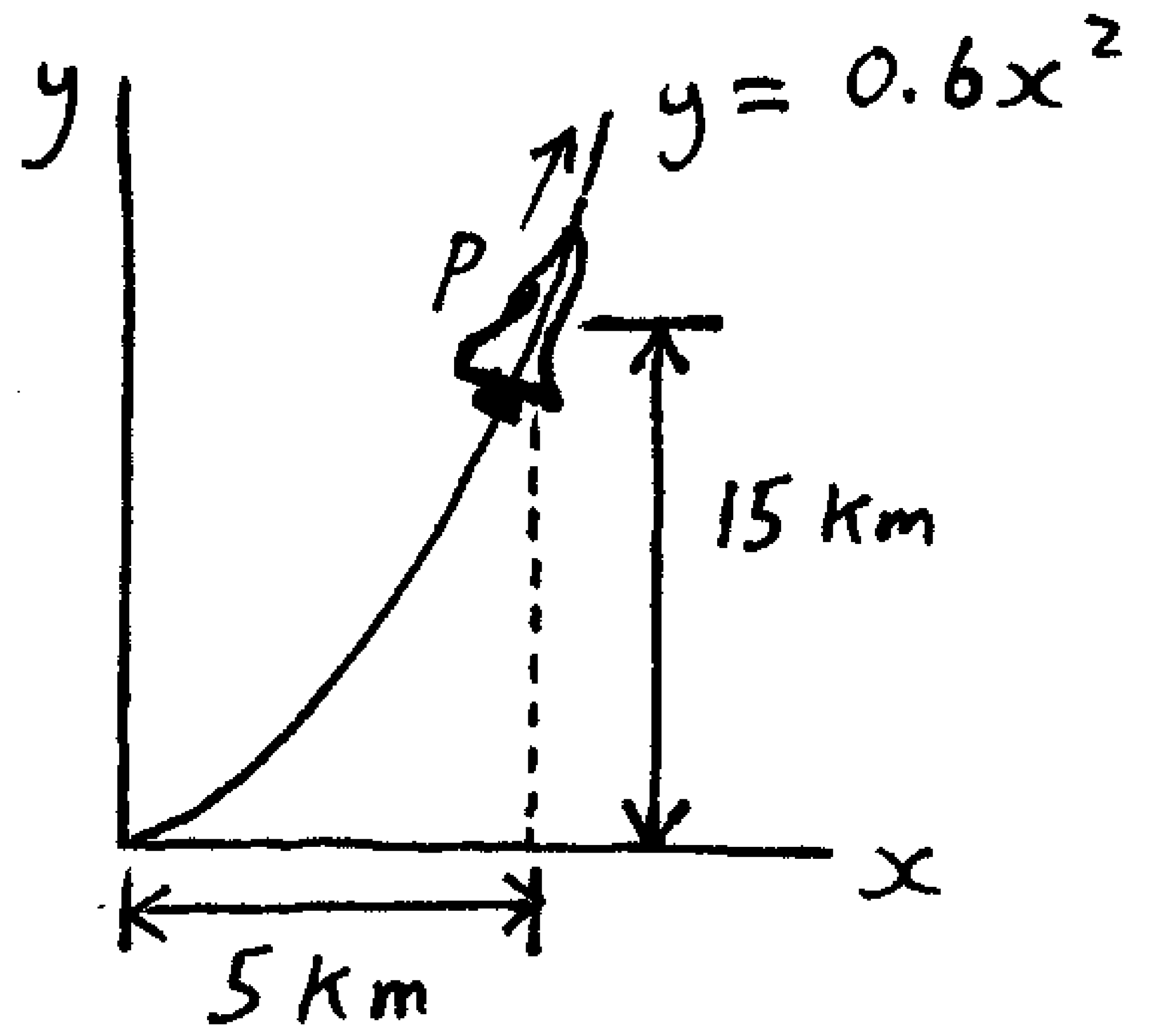
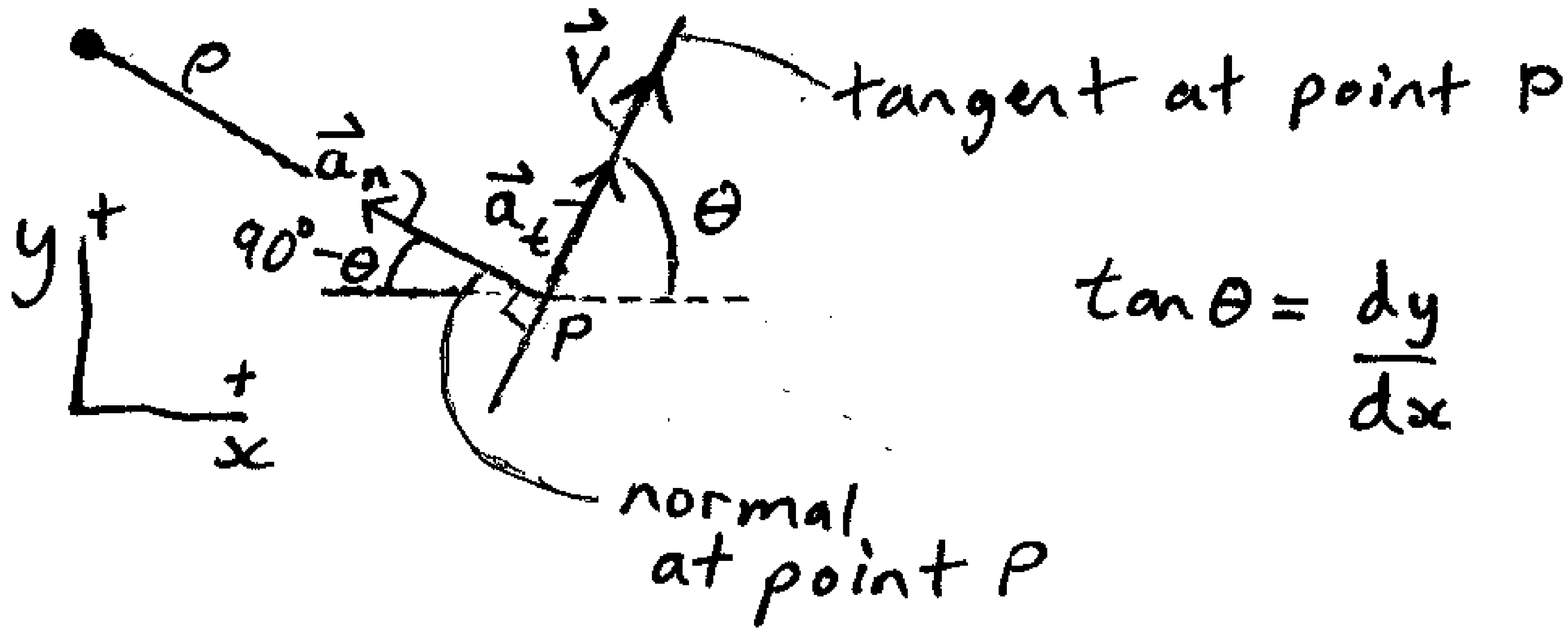
This is a curvilinear motion problem involving normal and tangential components (engineering mechanics).



A jet aircraft flies along a vertical parabolic path, and at point P it has a speed of 260 m/s, which is increasing at a rate of 0.9 m/s^2. Determine the magnitude of the acceleration at point P.

Solution:



$$\tan\theta = \frac{dy}{dx}$$

The magnitude of the normal acceleration is:

$$a_n = \frac{v^2}{\rho}$$ (a_n is the magnitude of the acceleration vector $\vec{a}_n$, normal to the arc at instant shown)

The magnitude of the tangential acceleration is:

$a_t = \frac{dv}{dt}$ (magnitude of the acceleration vector $\vec{a}_t$, tangent to the arc at the instant shown)

Now, $\tan\theta = \frac{dy}{dx}$

since, $y = 0.6x^2$

$$\frac{dy}{dx} = 1.2x$$

At $x = 5$ km, $\frac{dy}{dx} = 6$ (slope at point P)

Substitute: $\tan\theta = 6$, $\theta = \tan^{-1}(6) = 80.54°$

Next, $\rho = \left| \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right|$

$$\frac{d^2y}{dx^2} = 1.2$$

Substitute values for point P:

$\rho = \left| \frac{\left[1 + 6^2\right]^{3/2}}{1.2} \right| = 187.55$ Km (radius of curvature at point P)

$= 187.55 \times 10^3$ m

$v = 260\ m/s$ (speed of the aircraft, which is the magnitude of the velocity vector $\vec{v}$, tangent to the arc at the instant shown)

Then,

$$a_n = \frac{v^2}{\rho}$$

Substitute: $a_n = \dfrac{(260)^2}{187.55 \times 10^3} = 0.36\ m/s^2$

and, $a_t = 0.9\ m/s^2$

The vertical component of the resultant acceleration is:

$$a_y = a_n \sin(90° - \theta) + a_t \sin\theta$$

Substitute:

$$a_y = 0.36 \sin(90° - 80.54°) + 0.9 \sin(80.54°)$$

$$a_y = 0.947\ m/s^2$$

The horizontal component of the resultant acceleration is.

$$a_x = a_t \cos\theta - a_n \cos(90° - \theta)$$

Substitute:

$$a_x = 0.9 \cos 80.54° - 0.36 \cos(90° - 80.54°) = -0.207\ m/s^2$$

The magnitude of the acceleration is: $\sqrt{(-0.207)^2 + (0.947)^2}$

$= 0.97\ m/s^2$ (answer)