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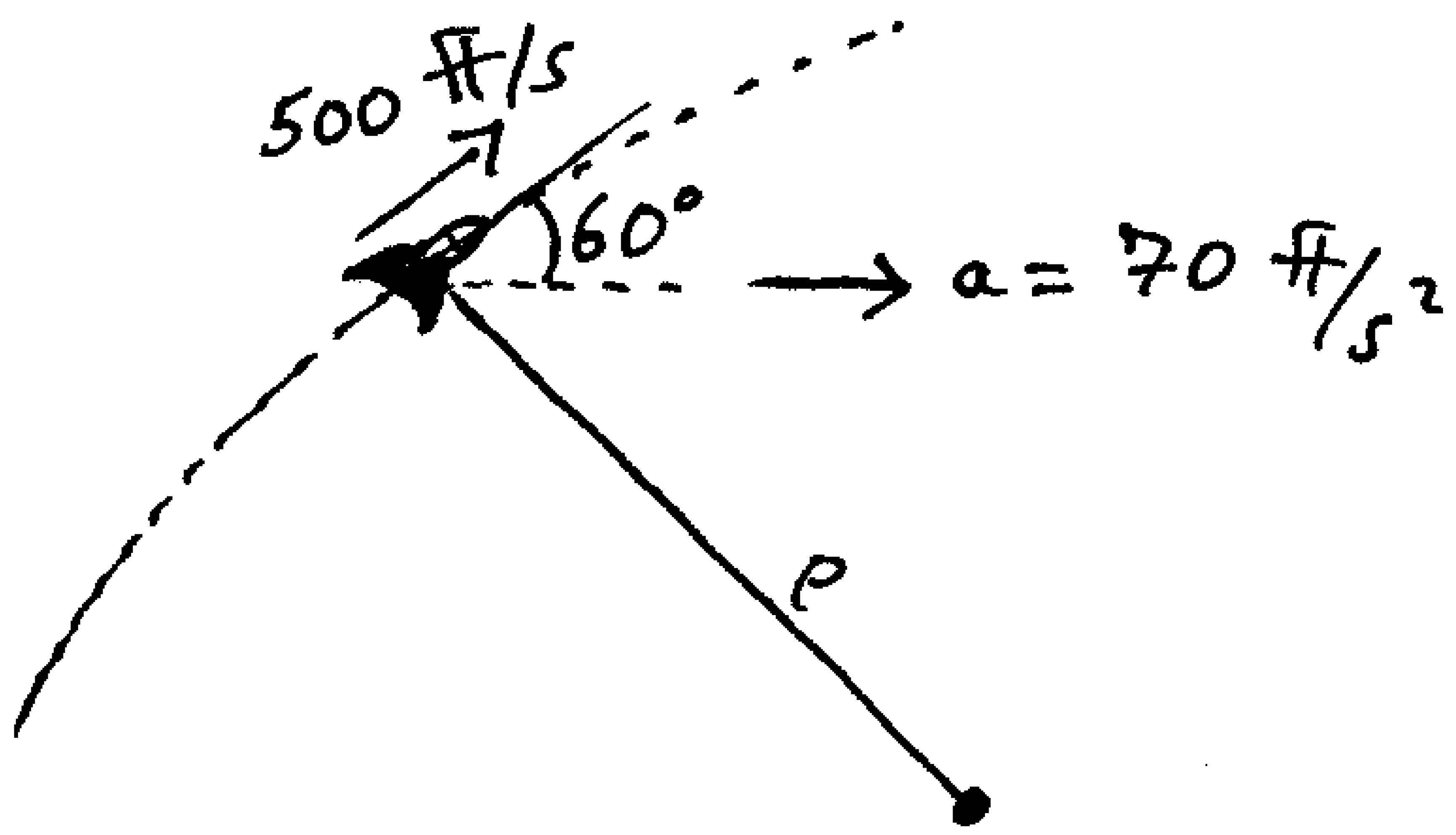
This is a curvilinear motion problem involving normal and tangential components (engineering mechanics).

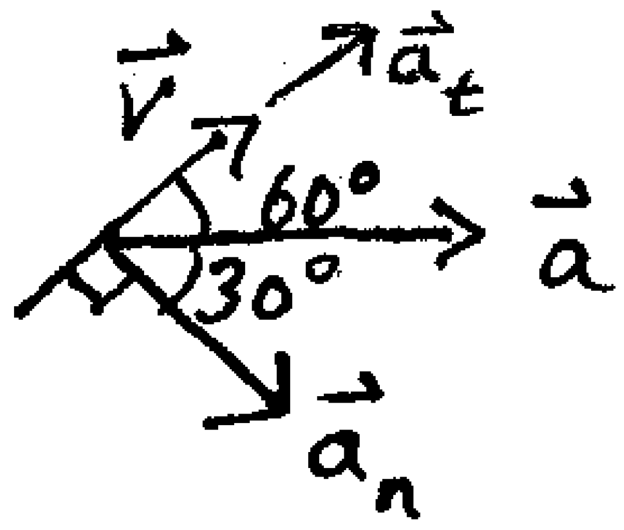


A jet aircraft is flying along an arc, and at the instant shown it has a speed of 500 ft/s and an acceleration of 70 ft/s² in the direction shown. What is the rate of increase in the speed of the plane, and the radius of curvature ρ of the arc, at the instant shown?

Solution:

The magnitude of the normal acceleration is:

$$a_n = \frac{v^2}{\rho}$$



(a_n is the magnitude of the acceleration vector $\vec{a}_n$, normal to the arc at instant shown)

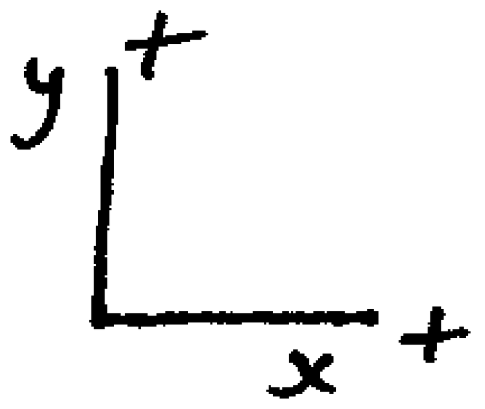
The magnitude of the tangential acceleration is:

$$a_t = \frac{dv}{dt}$$

(magnitude of the acceleration vector $\vec{a}_t$, tangent to the arc at the instant shown)

$v = 500 \text{ ft/s}$ (speed of the aircraft which is the magnitude of the velocity vector $\vec{v}$, tangent to the arc at the instant shown)

Choose the following sign convention, and orientation for xy axes.



Now,

$a_y = a_t \sin 60° - a_n \sin 30°$ (vertical component of resultant acceleration)

$a_y = 0$, since resultant acceleration vector $\vec{a}$ has no vertical component, with respect to the chosen sign convention

Therefore, $0 = a_t \sin 60° - a_n \sin 30°$ (1)

Next,

$a_x = a_t \cos 60° + a_n \cos 30°$ (horizontal component of resultant acceleration)

$a = |\vec{a}| = a_x = 70 = a_t \cos 60° + a_n \cos 30°$ (2)

($\vec{a}$ is the resultant acceleration vector)

Solve equations (1) and (2):

$a_t = 35 \text{ ft/s}^2$ (rate of increase of plane speed)

$a_n = 60.62 \text{ ft/s}^2$ (answer)

From before,

$$a_n = \frac{v^2}{\rho}$$

Substitute:

$$60.62 = \frac{(500)^2}{\rho}$$

$\rho = 4124$ ft (radius of curvature of arc)

(answer)