

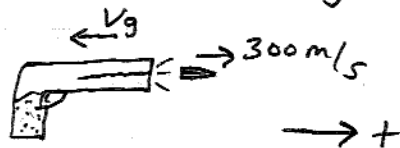
A bullet is fired out of a gun with a velocity of  $300 \text{ m/s}$ , with respect to ground. If the bullet has a mass of  $0.06 \text{ kg}$  and the gun has a mass of  $1.5 \text{ kg}$ , what percentage of the total kinetic energy (of the bullet and gun) does the bullet have?

Solution:

Use conservation of linear momentum equation.

The initial velocity of the bullet and gun is zero. So,

$$0 = 0.06 \times 300 - 1.5 v_g, \text{ where } v_g \text{ is the gun velocity after firing, with respect to ground}$$



Solving,  $v_g = 12 \text{ m/s}$

The kinetic energy of the bullet is

$$KE_b = \frac{1}{2} (0.06) (300)^2 = 2700 \text{ J}$$

The kinetic energy of the gun is

$$KE_g = \frac{1}{2} (1.5) (12)^2 = 108 \text{ J}$$

The percentage of  $\frac{\text{Kinetic energy}}{\text{total}}$  that the bullet has is:

$$\begin{aligned} \eta_b &= \frac{2700}{2700 + 108} \\ &= \underline{\underline{96.15\%}} \end{aligned}$$

Even though the bullet and gun experience the same magnitude of impulse, the bullet acquires much more of the kinetic energy

The bullet, as a result, can do a lot more damage if it hits something than the gun can (moving back at 12 m/s). In fact, the greater the mass of the gun, relative to the bullet, the greater the proportion of kinetic energy that the bullet has (after firing). To see this let's solve this problem again without using numbers.

Conservation of linear momentum:

$$0 = m_b v_b - m_g v_g$$

$$m_b v_b = m_g v_g \quad (1)$$

where  $m_b$  is the mass of the bullet,  $m_g$  is the mass of the gun, and  $v_b$  is the velocity of the bullet

Kinetic Energy ratio:

$$\eta_b = \frac{\frac{1}{2} m_b v_b^2}{\frac{1}{2} m_b v_b^2 + \frac{1}{2} m_g v_g^2}$$

$$\eta_b = \frac{1}{1 + \frac{m_g v_g^2}{m_b v_b^2}}$$

$$\eta_b = \frac{1}{1 + \frac{m_b}{m_g}} \quad \left( \text{after substituting equation (1)} \right)$$

$$\text{As } \frac{m_b}{m_g} \rightarrow 0 \quad \eta_b \rightarrow 100\%$$