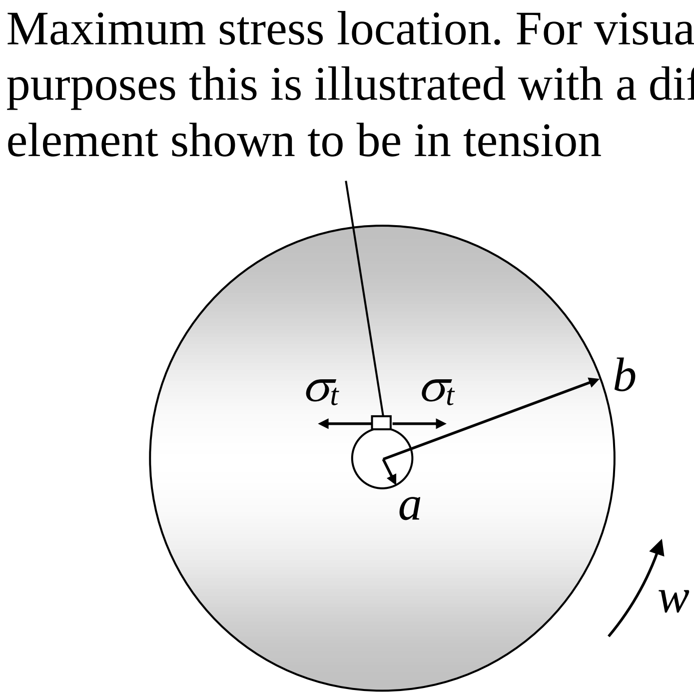
Can a standard CD-ROM drive shatter a CD?

A CD can be treated as an annular disk with inner hole radius *a* and outer radius *b*. The maximum stress occurs at the inner hole location at radius *a*. This is the part of the CD that will break first if it is spun too quickly. As the disk spins centripetal forces are generated inside the structure of the CD and these forces are what put the CD material under stress. If the stress is too great (due to the CD spinning too fast) the CD will break.



Assume the CD has no cracks or flaws in it, which would complicate the analysis.

The maximum stress of the rotating CD is located at the inner hole location (at radius *a*). This stress is tensile. At this location the stress is given by the following solid mechanics equation:

$$\sigma_t = \left(\frac{3+\nu}{4}\right)\rho w^2 b^2 \left(1 + \left(\frac{1-\nu}{3+\nu}\right)\frac{a^2}{b^2}\right)$$

Where:

σ_t is the tangential stress, which acts as tension in the circumferential direction. The maximum tensile yield stress which can be withstood by CD material (polycarbonate plastic) is roughly $65\text{x}10^6$ pascals (newtons per square meter).

ν is the Poisson's ratio for the CD material, which is 0.37

ρ is the density of the CD material, which is 1200 kg/m^3

w is the angular velocity of the CD, in radians per second. This quantity must be determined

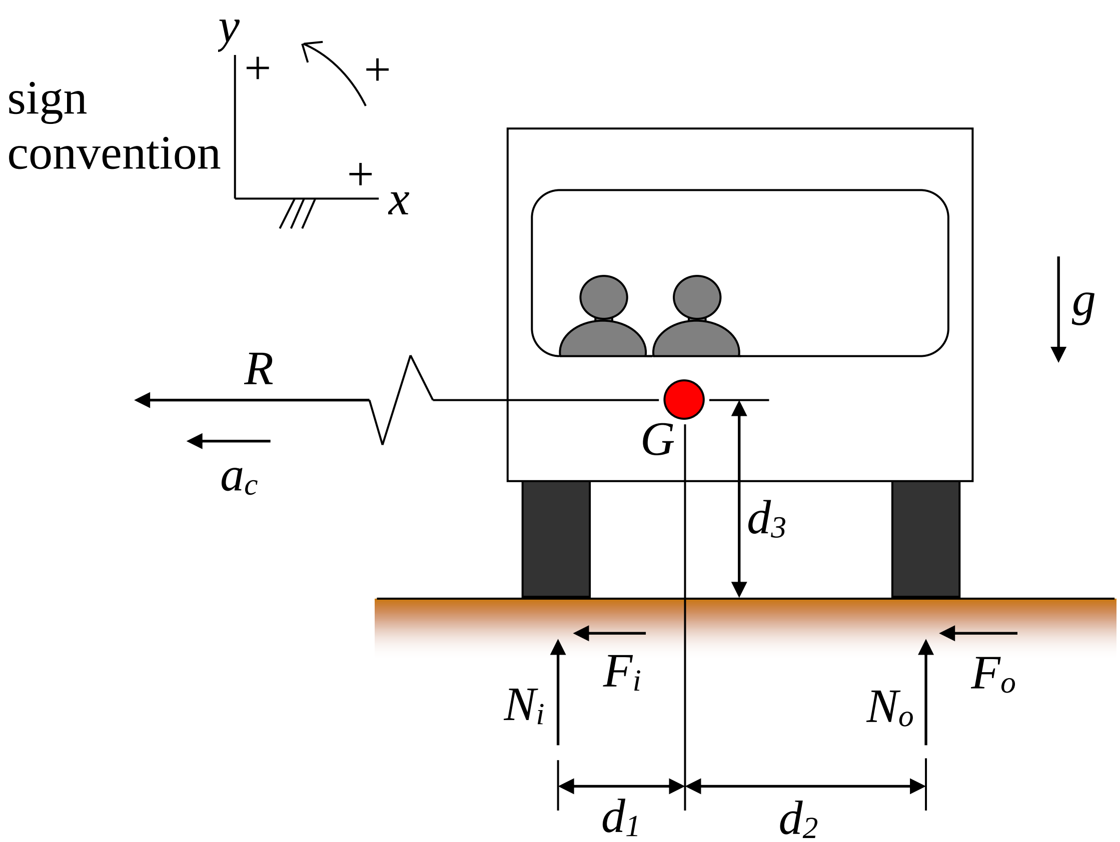
b is the outer radius of the CD, which is 0.06 m

a is the inner hole radius of the CD, which is 0.0075 m

Now solve for *w*. We get *w* = 4220 radians/second. This is equal to 40,300 RPM. This is well above the maximum rotational speed of CD-ROM drives which may top out at about 24,000 RPM. Therefore a standard CD-ROM drive *cannot* shatter a CD.

When a bus is moving at over 50 miles per hour, will moving passengers to the inside of the turn keep the bus from flipping?

This myth is taken from the 1994 movie *Speed*. To analyze this problem set up a free body diagram as shown below, with sign convention shown. As in the movie, the bus is traveling on a flat horizontal surface and going around a turn at speed *V*.



Where:

g is the acceleration due to gravity, which is 9.8 m/s^2

G is the center of mass of the bus + rider system, which is shifted to the left due to the passengers moving to the left of the bus

V is the speed of the bus at the center of mass location G

R is the radius of the turn, measured from the center of the turn to the center of mass G of the system

a_c is the centripetal acceleration of the center of mass of the bus + rider system

F_i is the horizontal force acting on the inside wheels, at the contact point with the ground

N_i is the normal force acting on the inside wheels, at the contact point with the ground

d_1 is the horizontal distance between the left wheel contact point (with the ground) and the center of mass G

d_2 is the horizontal distance between the right wheel contact point (with the ground) and the center of mass G

d_3 is the vertical distance between the ground and the center of mass G

F_o is the horizontal force acting on the outside wheels, at the contact point with the ground

N_o is the normal force acting on the outside wheels, at the contact point with the ground

Assume the bus is on the verge of tipping so that $N_i = 0$ and $F_i = 0$. Then find the turn radius R of the bus for a speed V of 50 mph. If the turn radius is greater than this value the bus will not flip over.

Treat this as a problem in two-dimensional rigid body dynamics. For simplicity assume the mass of the wheels is negligible. This allows the problem to be solved as a single rigid body, otherwise the wheels would have to be analyzed with separate equations which will not appreciably improve the accuracy of the solution.

Apply Newton's second law in the horizontal direction:

$$-F_o = ma_x$$

where m is the mass of the bus + rider system and a_x is the horizontal acceleration of the center of mass G.

Now, due to centripetal acceleration

$$a_x = -\frac{V^2}{R}$$

Where $a_x = a_c$. So the above equation for Newton's second law becomes

$$F_o = m\frac{V^2}{R} \quad (1)$$

Apply Newton's second law in the vertical direction:

$$N_o - mg = 0 \quad (2)$$

Assume the system is in a state of rotational equilibrium. This means there is zero torque acting on the system about the center of mass G, about an axis pointing out of the page. Mathematically we can write this as

$$N_o d_2 - F_o d_3 = 0 \quad (3)$$

(Note that we are ignoring three-dimensional dynamic effects in this equation. They are assumed to be negligible).

Solve for V from the above equations (1)-(3). We get

$$V = \sqrt{\frac{g d_2 R}{d_3}}$$

Assume a bus width of 2.6 meters and a center of mass height d_3 of 1.0 m. If we assume that G is in the center of the bus (corresponding to the passengers evenly distributed on both sides) then d_2 = 1.3 m. Since the 19 passengers altogether weigh much less than the bus, they would shift the center of mass only by a bit if they all moved to one side, so we can use d_2 = 1.3 m. For a bus speed of 22.35 m/s (50 mph) calculate the radius R. This is the minimum radius of the turn to avoid flipping over. We get R = 39 m.

As an exercise estimate the radius of the turn the bus made in the movie to see if it is at least 39 m.