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A car has a mass of 1700 Kg, a drag coefficient of 0.35, a rolling resistance coefficient of 0.01, a frontal area of $2 m^2$, and a horsepower of 150. What is the minimum time required for the car to go from 0 to 60 mph? What is the minimum time if the car has 300 hp, with all else the same?

Use density of air $= 1.2 \, Kg/m^3$, $g = 9.8 \, m/s^2$ and assume the car is on a perfectly flat surface, and assume that the wheels don't slip.

Solution:

The minimum time corresponds to the car using the entire horsepower during acceleration, and no mechanical friction acting internally in the car. This will result in a minimum acceleration time corresponding to maximum acceleration (from 0 to 60 mph).

which would reduce the horsepower delivered to the wheels, so that it falls below 150 hp, or 300 hp (the engine horsepower)

We can treat this as a simple one-dimensional problem.

By Newton's second law,

$\Sigma F = ma$, where m is the car mass, and a is the car acceleration.

$\Sigma F = F_c - F_d - F_r$ (sum of forces)

where F_c is the "push" force of the car wheels on the surface, driving the car forward

F_d is the drag force, due to air resistance

F_r is the rolling resistance of the car wheels on the surface

Now, $F_d = \frac{1}{2} C_d \rho A v^2$

where C_d is the drag coefficient
ρ is the air density
A is the frontal area of the car
v is the velocity of the car (we are assuming the air is stationary, no wind)

Now, $F_r = C_r m g$

where C_r is the rolling resistance coefficient

The power of the car is given by P, where

$P = F_c v$, $F_c = \frac{P}{v}$, P is constant

From before,

$\Sigma F = m a$

$\Rightarrow F_c - F_d - F_r = m a$

Substituting F_c, F_d, F_r, and $a = \frac{dv}{dt}$ we get

$$\frac{P}{v} - \frac{1}{2} C_d \rho A v^2 - C_r m g = m \frac{dv}{dt}$$

Thus,

(1) $$m v \frac{dv}{dt} + C_r m g v + \frac{1}{2} C_d \rho A v^3 = P$$

This is a differential equation which must be solved numerically for v as a function of time.

Substituting the known quantities into the above equation we get,

S.I. unit for power

$$1700 v \frac{dv}{dt} + 166.6 v + 0.42 v^3 = \underbrace{111855}_{150\ hp} \text{ (Watts)}$$

Initial condition:

$V_0(t=0) = 0$, but use $V_0 = 0.015$ m/s to avoid numerical problems

use forward integration (and time step of 0.0005 seconds)

we wish to find the time it takes for the car to reach 26.822 m/s (60 mph).

Solving, we find that time = 5.76 seconds

And if the car has 300 hp, then $P = 223710$ Watts and time for 0-60 mph is 2.74 seconds. This is about half the time!

Now, let's compare this to the case where air drag and rolling resistance (of the wheels) is ignored. From equation (1), $m v \frac{dv}{dt} = P$, which can be solved analytically:

$$Pt = \frac{1}{2} m v_f^2, \quad m = 1700\ kg, \quad v_f = 26.822\ m/s, \quad V_0 = 0$$

For $P = 111855$ W, $t = 5.47$ s   For $P = 223710$ W, $t = 2.73$ s

These are very close to the times calculated when accounting for air drag and rolling resistance, especially for the higher horsepower case of 300 hp!

Therefore, the simple equation $Pt = \frac{1}{2} m v_f^2$ is a good approximation in many instances, as long as the following terms (from equation (1)) are much less than P (for the range of v considered):

$$C_r m g v$$

$$\frac{1}{2} C_d \rho A v^3$$

which means that they contribute little (in equation (1)).

In fact, the assumption of constant power (P) for an accelerating vehicle is realistic and students will appreciate being able to solve practical problems that correspond well to their experiences.