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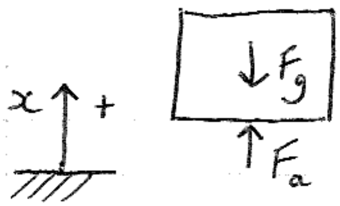
During a bench press, does the amount of work, or power, required depend on the rate at which the weight is lifted?

Solution:

Draw a free body diagram of the weight, along with sign convention.



where x is the $\overbrace{\text{lift distance}}^{\text{for one repetition}}$, F_g is the force of gravity, and F_a is the applied force exerted by the lifter.

$F_g = mg$, where m is the mass of the weight and g is the acceleration due to gravity

By Newton's second law applied to the weight:

$F_a - F_g = ma$, where a is the acceleration of the weight

Thus, $F_a = ma + F_g = ma + mg$ (1)

The work required is given by the integral equation:

$W = \int_0^T F_a \, dx$, where T is the lift time and dx is the incremental lift distance.

Substitute (1) into the above equation:

$$W = \int_0^T (ma + mg)\,dx = m\int_0^T a\,dx + mg\int_0^T dx$$

Now, $a = \frac{dv}{dt}$, where v is the velocity of the weight

Therefore, $W = m\int_0^T \frac{dv}{dt} \cdot dx + mg\int_0^T dx$

$$W = m\int_0^T dv \cdot \frac{dx}{dt} + mg\int_0^T dx$$

$\frac{dx}{dt} = v$, hence,

$$W = m\int_0^T v \cdot dv + mg\int_0^T dx$$

Note:
$x = x(t)$
$v = v(t)$,
where t is time

Therefore,

$$W = m \cdot \frac{1}{2}v^2(T) - m \cdot \frac{1}{2}v^2(0) + mgx(T) - mgx(0)$$

$v(0)$ is the weight velocity at the bottom of the lift. $v(T)$ is the weight velocity at the top of the lift. $v(0) = v(T) = 0$

Therefore, $W = 0 + mg\underbrace{[x(T) - x(0)]}_{= \text{lift distance} \equiv L}$

Hence, the work required to lift the weight is $W = mgL$, which is <u>constant</u> and <u>independent</u> of the rate at which the weight is lifted.

Now, power $= F_a v = m(a+g)v$. This is an instantaneous quantity and it <u>does</u> depend on how fast the weight is lifted at any one time.