During a bench press, does the amount of work or power required depend on the rate at which the weight is lifted?

Solution:

Draw a free body diagram of the weight, along with sign convention.

\[ \begin{array}{c}
\uparrow F_a \\
\downarrow F_g \\
x \uparrow + \\
\hline
\end{array} \]

For one repetition, where \( x \) is the lift distance, \( F_g \) is the force of gravity, and \( F_a \) is the applied force exerted by the lifter.

\[ F_g = mg, \text{ where } m \text{ is the mass of the weight and } g \text{ is the acceleration due to gravity.} \]

By Newton's second law applied to the weight:

\[ F_a - F_g = ma, \text{ where } a \text{ is the acceleration of the weight} \]

Thus, \[ F_a = ma + F_g = ma + mg \] (1)

The work required is given by the integral equation:

\[ W = \int_0^T F_a \, dx, \text{ where } T \text{ is the lift time and } dx \text{ is the incremental lift distance.} \]

Substitute (1) into the above equation:

\[ W = \int_0^T (ma + mg) \, dx = m \int_0^T a \, dx + mg \int_0^T dx \]
Now, \( a = \frac{dv}{dt} \), where \( v \) is the velocity of the weight.

Therefore, \( W = m \int_0^T \frac{dv}{dt} \cdot dx + mg \int_0^T dx \)
\[ W = m \int_0^T v \cdot dv + mg \int_0^T dx \]
\[ \frac{dx}{dt} = v, \text{ hence,} \]
\[ W = m \int_0^T v \cdot dv + mg \int_0^T dx \]

Note:
\[ x = x(t) \quad v = v(t), \text{ where } t \text{ is time} \]
\[ W = m \cdot \frac{1}{2} v^2(T) - m \cdot \frac{1}{2} v^2(0) + mg x(T) - mg x(0) \]
\[ v(0) \text{ is the weight velocity at the bottom of the lift. } v(T) \text{ is the weight velocity at the top of the lift. } v(0) = v(T) = 0 \]

Therefore, \( W = 0 + mg \left[ x(T) - x(0) \right] \)
\[ \text{= lift distance} \equiv \ell \]

Hence, the work required to lift the weight is \( W = mg \ell \), which is constant and independent of the rate at which the weight is lifted.

Now, power = \( P = F_a v = m(a + g)v \). This is an instantaneous quantity and it does depend on how fast the weight is lifted at any one time.