1.

Did Hungarian archers get twice the penetration shooting a bow from a galloping horse?

Getting twice the penetration would imply that the kinetic energy of the arrows approximately doubled when shooting them from a horse. Doubling the kinetic energy would approximately produce twice the penetration since (Kinetic energy) = (Work done during penetration of the target) = (penetration force)x(penetration distance). This assumes the penetration force is roughly constant, as a first estimate. And with penetration force constant the penetration distance would double for a doubling of the kinetic energy.

A galloping horse runs at about 14 m/s. This speed would be added to the arrow speed (measured when shot from the ground). The arrow speed would have to be such that the kinetic energy of the arrow, when shooting from a horse, is twice the kinetic energy when shooting from the ground. For the arrow shot from the ground, the kinetic energy is equal to \((1/2)mV^2\), where \(m\) is the mass of the arrow and \(V\) is the arrow speed. For the arrow shot from the horse, the kinetic energy is equal to \((1/2)m(V+14)^2\). We can solve for \(V\) by setting the ratio of these kinetic energies equal to 2. We have \((V+14)^2/V^2 = 2\). Solve for \(V = 34\) m/s (76 mph). If the arrow speed of the Hungarian archers was close to this speed then the myth is at least plausible.
2.

In archery, when an arrow is released it can oscillate during flight. If we know the location of the center of mass of the arrow \((G)\) and the shape of the arrow at an instant as it oscillates (shown below), we can determine the location of the nodes. The nodes are the “stationary” points on the arrow as it oscillates.

Using a geometric argument (no equations), determine the location of the nodes.

Assume that the arrow oscillates in the horizontal plane, so that no external forces act on the arrow in the plane of oscillation.

Solution

The key piece of information here is that no external forces are acting on the arrow in the plane of oscillation. Therefore, for purposes of solving this problem we can treat the center of mass \(G\) as stationary, even though the arrow itself is oscillating. This becomes evident by Newton’s Second Law, where \(\sum F_{\text{ext}} = ma\). In the plane, \(\sum F_{\text{ext}} = 0\), so \(a_G = 0\). This means that the center of mass \(G\) of the arrow is either moving at a constant horizontal velocity in a straight line, or \(G\) is stationary. Although the former is true, for simplicity purposes we can treat the center of mass \(G\) as stationary since it will help us visualize what is happening.
The first step is to draw a line $L$ passing through $G$ and aligned with the direction of motion of the arrow. This line is also the symmetry line of the arrow as it oscillates.

The second step is to reflect the arrow about this symmetry line. The nodes are the points of intersection of the arrow (shown above) with the reflected arrow. We are done.